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ANALYTICAL THEORY OF THE CAMPINI PROPULSION SYSTEM

By S. Campini

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Page 10, line 10: Cp = 0.236 should read $C_p = 0.236$.

Page 17, formula (3): Td should be T_d.

Page 23, denominator of formula (5): ν^2 should be v^2.

Page 24, formula third line from bottom should read:

$$\frac{W'}{E\ C_t} = \psi\ B = \psi\ \frac{e\ \gamma + X\ [\varphi(1 + k) - e\ \gamma]}{(1 + k)} + t$$

Page 25, formula ninth line from bottom should read:

$$R^2 = \frac{V^2}{v^2}$$

Page 26, formula (4): R^2 should be R.

Page 43, formula ninth line from top should read:

$$R^2 = z\ \psi\ \theta\left(1 + \frac{\varphi\ r\ K}{a\ e\ \gamma}\right)$$

Page 45, formula (1): w should be ω.

Page 47, formula (4) should read:

$$\boxed{r = r_o\ \frac{v_o}{v}\ \frac{T}{T_o}}$$

Page 49, formula near bottom of page: Remove n from square root sign.

On pages 22 and 23, the formulas containing α were intended to use the letter a, judging by the second part of the article. The confusion is apparent in the first two formulas on page 47.

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ANALYTICAL THEORY OF THE CAMPINI PROPULSION SYSTEM*

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SUMMARY

Following the description of the new propulsion system and the definition of the propulsive efficiency, this efficiency is calculated under various conditions of flight with allowance for all internal losses.

The efficiency and consumption curves are plotted, their practical values discussed and the behavior of the system analyzed at various altitudes and speeds. The superiority of this over the conventional engine-propeller system, starting from 400 kilometers per hour (248 mph), with respect to range and weight per horsepower output is affirmed.

The immediate possibilities of the new system in flight at high and very high altitudes in relation to the theoretical and experimental results obtained are discussed in detail.

The present report is a continuation of the study made in 1929 of which the first two parts were published in 1930 (reference 1), while the remaining parts dealing with the action of jet propulsion in fluid medium have never been released.

Since that time the theoretical studies have been combined with systematic experimental research for the purpose of checking the practical value of its fundamental laws and for the experimental determination of the factors which would enable us to pass safely from the theoretical to the practical stage.

These experiments, made in collaboration with the Caproni Airplane Company of Milan, have confirmed all of afore-mentioned laws, and afforded a marked improvement in the practical value of the various factors defining this

*"Sulla Teoria Analitica del Moto-Propulsore Campini."
L'Aerotecnica, vol. XVIII, no. 1, Jan. 1938, pp. 18-63.

type of propulsion. (An elaborate report on these factors is to be published in due course.)

The present report deals exclusively with the particular jet-propulsion system mentioned in the cited reports. The discussion is limited, for the present, to the analytical study of the efficiency and the consumption for the case of application to aircraft and to the plotting of the practical operating curves secured theoretically by the use of suitable experimental factors.

The purpose of this study is to enable aeronautical technicians, interested in problems related to high-speed flight at low and high altitudes, to calculate directly and rapidly the results that can be obtained in practice with this system of propulsion when applied to any apparatus in accordance with the characteristics required, while using conventional heat engines and compressors.

1. CHARACTERISTICS OF THE PROPULSION SYSTEM

The description of the system can be limited to the operational part and specifically to the extent necessary to secure an accurate solution of its efficiency (reference 2).

The propulsion system creates a thrust reaction in the following manner (reference 3).*

The fluid medium enters under the effect of a simple relative motion, that is, without deflection or shock, and, as soon as it enters the intake duct, it is subjected first to a dynamic compression by reason of the quasi-integral transformation of the kinetic energy of inflow into pressure, then to a further mechanical compression by means of the compressor driven from a heat engine. Subsequently this twice compressed fluid is heated at constant pressure by thermic means so that the speed in the expansion of the discharge is augmented to the amount necessary to produce the desired propulsive energy.

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*The procedure was developed and explained by René Lorin as early as 1913. The calculation of the efficiency was made by G. S. Steckin in 1924 (see references 3 and 19 for greater details), while Campini deserves much credit for developing these ideas.

In practice the thermic means involve primarily the utilization of the heat dissipated by the engine and secondarily the eventual direct combustion of atomized and vaporized fuel.

The operational definition of the system is shown in figure 1. The air entrained by relative emotion enters duct 1 where its kinetic energy derived from the relative speed of inflow, is changed to pressure. Following this preliminary compression accompanied by a marked decrease in speed the air passes through compressor 2 where its pressure is again considerably increased. It then receives the heat dissipated by the engine 3, which drives the compressor and subsequently receives the eventual calories produced by the burners 4. Lastly it is expanded in nozzle 5, where the thermodynamic energy accumulated in the various phases of compression and heating changes to speed of discharge.

2. CALCULATION OF THRUST AND RESISTANCE

If the propulsive system as described and illustrated in figure 1, were isolated, that is, had none other than a propulsive function, the thrust produced by it would be equal and opposite to the momentum impressed by it on the ambient fluid, as is known from the momentum theory (references 1 and 3). In this case, by defining two control sections 1 and 2, the first upstream, the second downstream from the propulsive system where the fluid filaments become parallel again and in which the relative pressures are zero, the foregoing theorem would give:

$$S_u = \rho \int_{\infty} v_1 \ (v_2 - v_1) \ d \ A_1 \tag{1}$$

where ρ = density, $v_2 - v_1$ the increment of the speed incurred by a general flow filament which separates the area dA on section 1 while passing from section 1 to section 2. But since the propulsive unit cannot be isolated because it also has the function of fuselage and because of the practice obtaining in aeronautical science of never considering the resistance offered by the engine and its cowling as a negative portion of the thrust produced by the propulsion unit, it is preferrable in this case also to distinguish between an external drag of the unit and a thrust produced by the same.

The external drag, except in special cases, can thus be included with the various resistances encountered on aircraft and hence be included in the aerodynamic characteristics of the model; it can be ascertained direct in wind-tunnel tests, the difficulties regarding inlet and discharge nozzles being solved experimentally, or else calculated.

The effectuated distinction calls for a precise definition of thrust and resistance. Let us assume as external resistance Re of the system, that is by definition, the integral of the impulses, undergone by the fluid not passing through the unit, between control sections 1 and 2. Assume further as thrust S the same integral referred to the fluid passing through the unit. By definition, calling ω the area isolated from control surface 1 by the fluid that passes through the unit, the momentum equation reads:

$$S = \rho \int_{\omega} v_1 \ (v_2 - v_1) \ d \ A_1 \tag{2}$$

$$R_e = \rho \int_{\infty - \omega} v_1 \ (v_1 - v_2) \ d \ A_1 \tag{3}$$

The effective thrust S_u of the isolated propulsion unit therefore follows as the difference between thrust and external resistance of the system.

From the definition of the external resistance it is then deduced that, in the absence of viscosity, the application of d'Alembert's theorem to the indefinite solid constituted by the propulsion unit and the fluid jet that enters and discharges from the unit, immediately affords:

$$R_e = \rho \int v_1 \ (v_1 - v_2) \ d \ A_1 = 0$$

However, Re is positive in viscous fluid and its exact value must be secured experimentally for each case.

It is interesting to compare the external resistance and the resistance of the moving body obtained from the propulsion unit by closing the inlet and outlet ports by means of aerodynamically correct surfaces which transform the propulsion unit into a streamline body.

Experiments of this nature have been made on solids of revolutions in the Caproni wind tunnel and repeated in the Rome tunnel with identical results. Similar experiments have been conducted by the NACA (references 5 and 6) on jet-propulsion systems with very small discharge areas as compared to diametral section.

Theoretically, it is found that for ratios between diameter of intake orifice and diameter of maximum outside section in excess of 1/3 to 1/2 a considerable decrease in external resistance becomes manifest on opening of the ports, while for ratios below 1/10 the effect of the ports is negligble (references 5 and 6).

This is obvious, moreover, since the body drag must depend largely on the difference between the diameters of the maximum outside section and that of the intake.

These experiments and their interpretation will be discussed later on. For the present it is simply stated that in practice for ratios of around one-half the body drag is so low that it can be included in the friction drag with a correction factor equal to 1.05 to 1.10.

The external drag can thus be computed by means of the formula known in aerodynamics for the solution of the turbulent friction of a flat plate:

$$R_e = c_t \; q \; \sigma_e \quad (1.05 \text{ to } 1.10) \tag{4}$$

where q is the dynamic pressure (mm), σ_e the outside area and c_t the friction drag, in relation to the Reynolds number relevant to the speed of the test per length of propulsive unit, a factor which can be expressed by the Kármán relation.*

$$c_t = 0.072 \sqrt[5]{\frac{1}{Ry}} \qquad Ry = \frac{v\,l}{\mu} \tag{5}$$

As for the thrust, it can be obtained at once from (1), when the speeds in control sections 1 and 2 are constant; an assumption which in practice is completely verified for the inflow velocity and almost completely for the discharge velocity.

* Verified in the case in point at $Ry = 10^6$.

With ω denoting the area isolated by control section 1 from the fluid jet which enters the propulsion unit and ρ the density and v and V the forward speed and the relative speed of discharge, respectively, we get:

$$S = \omega \rho v (V - v) \tag{6}$$

Allowing for the fact that in such propulsion unit the mass flowing out is greater than that flowing in, because of the fuel being united with the fluid, and calling α the quantity by weight inducted by the unit and ϵ the quantity of fuel mixed per kilogram of indicated air, we have, by putting $Q = \omega \rho g v$

$$S = \frac{Q}{g} (\alpha V - v) \text{ with } \alpha = 1 + \epsilon \tag{7}$$

This formula, which has been frequently verified experimentally and which with $\alpha = 1$ gives the thrust of an equivalent propeller, permits the direct calculation of the thrust, once quantity Q, the actual mean rate of discharge and the coefficient of increase of mass α are known.

3. EFFICIENCY OF THE PROPULSION SYSTEM

The propulsive efficiency is defined as the ratio of work done by the thrust S and the mechanical equivalent of heat energy absorbed per unit time.

With C denoting the calories consumed per kilogram of ambient fluid the propulsive efficiency follows at

$$y = \frac{S v}{Q . C E} = \frac{v (\alpha V - v)}{C E g}$$

But, bearing in mind that the jump in speed $V - v$ between control sections 1 and 2 requires for each kilogram of fuel the production of kinetic energy

$$W = \alpha \frac{V^2}{2 g} - \frac{v^2}{2 g}$$

it follows immediately, putting for formal simplicity:

$$\frac{V}{v} = R \qquad \boxed{m = \frac{2\,(\alpha\,R - 1)}{\alpha\,R^2 - 1}} \tag{8}$$

that the propulsive efficiency of the system can be expressed in the form

$$\boxed{y = m\,\frac{W}{E\,C}} \tag{9}$$

This formula obviously holds good for any type of propulsion system including the engine-propeller system.

Factor m, very appropriately termed the propulsive efficiency by some writers, is independent of the type of propulsion system, but depends entirely upon the ratio of the relative speed upstream and downstream from the system. Factor $W/E\,C$, on the other hand, characterizes the propulsion unit and may be termed the internal efficiency of the system itself.

In the case of propeller propulsion, with η called the propeller efficiency, the efficiency evidently becomes

$$y = \eta\ e$$

where e represents the efficiency factor of the engine, that is, the ratio of energy input at the propeller shaft to the mechanical equivalent of the consumed heat energy. The term η is then given, of course, by the product of propulsive efficiency m and the blower efficiency of the propeller (reference 7, p. 445 sec. 64).

Since $\alpha = 1$ in this instance, we have:

$$m = \frac{2\,v}{V + v} = \frac{2}{R + 1}$$

The engine efficiency is replaced in modern engineering by the specific gasoline or fuel oil consumption. This could also be done in the case in question by an artifice, but because of the interference between propulsive and thermal efficiency it is more logical to refer to a consumption per horsepower per hour of the propulsive power rather than to the power produced as kinetic energy W. Hence, the propulsive efficiency of any propulsion system in which the engine gives a fuel and oil consumption of π grams per horsepower per hour of propulsive power output and in which the mean heat value of fuel and oil is denoted by P, reads:

$$y = \frac{3600 \times 75}{\pi P E} \qquad \pi = \frac{3600 \times 75}{y P E}$$

To illustrate:

For $P = 11{,}000$ (gasoline) we get $\pi = 57.50/y$ in gr/hp/hr X.

A modern propulsion unit in which the engine consumes 230 grams of gasoline and 10 grams of oil and the mean propeller efficiency is 0.8, would have a consumption of 320 grams per horsepower and a propulsive efficiency of 0.19.

On the basis of the foregoing a comparison can then be established forthwith between different propulsion systems.

The calculation of the efficiency of the propulsive system then reduces to that of the propulsive efficiency m and of the internal efficiency W/E C and hence of the discharge velocity V which defines the consumption C for the velocity v under consideration.

The calculation can be made rather easily in this instance, by following the flow in its thermal evolutions and, with some care even some relatively small analytical expressions can be assumed.

Therefore let us examine separately, and in order, the various evolutions experienced by the fluid in relation to the characteristics of the compressor and of the engine with due regard to all the losses produced within the system.

4. DYNAMIC COMPRESSION

If T_0 is the absolute temperature of the ambient air and v speed of advance of the propulsion system, the compression effectuated in the intake tube, in the case of complete stoppage and in absence of shock and friction-losses, follows from the aero-thermodynamic relation which gives the dynamic pressure of a flow under adiabatic compression:

$$\frac{v^2}{2g} = \frac{\chi R T_0}{\chi - 1}\left[\left(\frac{p_1}{p_0}\right)^{\frac{\chi - 1}{\chi}} - 1\right]$$

But instead of conputing the dynamic pressure for obtaining somewhat general results comparatively suitable for a comprehensive treatment, it is more convenient to follow the variations in absolute temperature (references 3 and 12).

This eliminates the factor with the fractional component which otherwise complicates the determinations.

For this purpose a brief consideration of the aerodynamics involved may be timely: it is well known (reference 9) that in compression or expansion without exchange of heat with the outside, but with eventual development of heat due to shock and friction, the conservation of energy is expressed by the fundamental hydrodynamical equation.

$$d\,W = \frac{v\,d\,v}{g} = -\frac{d\,p}{\rho\,g} - d\,R \qquad (1)$$

where $\frac{d\,p}{\rho\,g}$ is the work done per kilogram of fluid against the pressure and $d\,R$ the energy against the forces of friction and shock, energy which is transformed into heat, This equation, which is fundamental for the study in question can be changed, of course, by making use of the first principle in its form of $d\,Q = c_v\,d\,T + Ap\,d\,V$ and integrating with c_p constant, in the fundamental equation

$$\boxed{\frac{\Delta\,W}{E c_p} = -\,\Delta\,T} \qquad (2)$$

which permits direct deduction of the finite increments or decrements $\Delta\,T$ of the temperature corresponding to any finite variation $\Delta\,W$ of the kinetic energy for transformation, whether adiabatic or with increase in entropy, or adiabatic or isentropic. The effect of friction manifests itself evidently in requiring, by equal variations in kenetic energy, a different value of $d\ p/\rho\ g$ and hence a variation of the pressure different from that necessary with an adiabatic change.

Following this brief discussion we can proceed at once with the calculation of the increment ΔT due to the dynamic compression.

Bearing in mind that $E = 427$ and $Cp = 0.236$ for air and temperature within -50° to 600° and denoting the residuary velocity after the change with Vr the foregoing equation affords:

$$\boxed{\Delta T_d = z \frac{v^2}{2\ g\ Ec_p} \text{ with } Z = 1 - \left(\frac{v_r}{v}\right)^2} \qquad (3)$$

If T_o is the ambient temperature it follows that the temperature after the transformation, whether accompanied by friction or not, is in every case:

$$T_1 = T_o + \Delta T_d = T_o + z \frac{v^2}{1980}$$

and if the change takes place without friction, that is, adiabatically, it would afford a compression ratio attainable from the known relation

$$\frac{p'_1}{p_o} = \left(\frac{T_1}{T_o}\right)^{\frac{\chi}{\chi-1}} \qquad (4)$$

If, in this latter case, a new adiabatic expansion is produced it again would yield the initial kinetic energy. However, because of the friction, the pressure reached is less than p'_1, so with θ_p denoting the efficiency of the pressure of the transformation duct this effective pressure p_1 follows the relation

$$\boxed{p_1 - p_o = \theta_p\ (p'_1 - p_o)} \qquad (4')$$

where the term $p_i - p_o$ can be secured experimentally by means of a pressure gage hence, with $p_1 - p_o$ known the efficiency θ_p of the pressure can be deduced.

The thermodynamic potential energy actually accumulated is that which can be obtained by complete adiabatic expansion, starting from the effective pressure p_o and terminating with the outside pressure p'_o. The effective temperature jump ΔT_u and the final temperature are easily obtained; with T_o denoting the terminal temperature of expansion the effective jump ΔT_u is according to equation (4):

$$\Delta T_u = T_1\left(1 - \frac{T'_o}{T_1}\right) = T_1\left[1 - \left(\frac{p_o}{p_1}\right)^{\frac{\chi-1}{\chi}}\right]$$

It follows that the thermodynamic efficiency of the change of kinetic energy into pressure taking place in the intake duct is:

$$\boxed{\frac{\Delta T_u}{\Delta T_d} = \theta = \frac{1 - \left(\frac{p_o}{p_1}\right)^{\frac{\chi-1}{\chi}}}{1 - \left(\frac{p_o}{p'_1}\right)^{\frac{\chi-1}{\chi}}}} \tag{5}$$

For extremely low compression the development in series, with the powers greater than of the first order discounted, affords:

$$\theta = \frac{p'_1}{p_1}\,\theta_p \begin{cases} d\,p' = p'_1 - p_o \\ d\,p = p_1 - p_o \end{cases}$$

For high compression it is necessary to employ the foregoing formula reproduced in the graph of figure 2. It is readily seen that the thermodynamic efficiency is always superior to that of pressure θ_p and tends to unity whatever the value of θ_p (the only condition being: $\theta_p \neq 0$).

This accords with formula (5) in which θ always equals 1 for $\frac{p_o}{p'_1} = 0$ and which has its reason in the fact that for extremely high operating speeds the shock and friction energy is not lost at all but becomes heat which in a large measure is utilizable in the subsequent re-expansion. The effective ratio of dynamic pressure with friction results in the conclusion given by:

$$\frac{p_1}{p_o} = \left(\frac{T_1}{T'_o}\right)^{\frac{\chi}{\chi-1}} \text{ with } T'_o = T_1 - \Delta T_u = T_1 - \theta \Delta T_d \quad (6)$$

in function of the thermodynamic efficiency θ. Moreover, the kinetic energy which could be produced with a new adiabatic expansion, without loss, follows from (3) and (1) at

$$\frac{v_o'^2}{2 g E c_p} = \theta \Delta T_d + \frac{v_r^2}{2 g c_p E} = z \theta \frac{v^2}{2 g E c_p} + \frac{v_r^2}{2 g E c_p} \quad (7)$$

5. MECHANICAL COMPRESSION

This compression acts on the air at the absolute pressure and temperature given in the preceding paragraph; the air, moreover, still has the residuary kinetic energy for each kg $\frac{v_r^2}{2 g}$.

Suppose the compression raises the absolute pressure to value P while the residuary velocity v_r at the compressor outlet remains unaltered. In the absence of friction losses this compression would raise the absolute temperature in the ratio:

$$\frac{T'_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\chi-1}{\chi}} \quad (1)$$

resulting in an increment of temperature of

$$\Delta T_m = T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\chi-1}{\chi}} - 1\right] \quad (2)$$

whence, after compression, the theoretical temperature would be:

$$T'_2 = T_1 + \Delta T_m \quad (3)$$

Accordingly the effective ratio of compression p_2/p_1 is dependent upon the theoretical increment ΔT_m conformably to equation (4) section 5.

$$\frac{p_2}{p_1} = \left(\frac{T'_2}{T_1}\right)^{\frac{\chi}{\chi-1}} \qquad (3')$$

But, since the pressure efficiency of the compressor is less than unity, it follows that in order to obtain pressure p_2 it is necessary to prevent a theoretical pressure p'_2 linked with the real p_2 through

$$\boxed{p_2 - p_1 = \gamma_p \, (p'_2 - p_1)} \qquad (4)$$

where γ_p is the pressure efficiency of the compressor.

But in correspondence with this theoretical pressure p'_2 a rise in the effective temperature

$$\Delta T_{me} = T_1\left[\left(\frac{p'_2}{p_1}\right)^{\frac{\chi-1}{\chi}} - 1\right] = \frac{\Delta T_m}{\gamma} \qquad (5)$$

where γ = adiabatic efficiency of compressor, is necessary according to the foregoing.

Hence in order to obtain the effective pressure p'_2 required per kilogram of air, a compression energy equal to

$$\boxed{\Delta W_m = c_p \, E \, \Delta T_{me} = c_p \, E \, \frac{\Delta T_m}{\gamma}} \qquad (6)$$

is necessary.

The foregoing adiabatic efficiency is then associated with the pressure efficiency γ_p through

$$\boxed{\gamma = \frac{\left(\frac{p_2}{p_1}\right)^{\frac{\chi-1}{\chi}} - 1}{\left(\frac{p'_2}{p_1}\right)^{\frac{\chi-1}{\chi}} - 1}} \qquad (7)$$

which for low pressures as encountered in practice, is, after development in series,

$$\gamma \cong \gamma_p \frac{1 - 0.355 \text{ d } p/p_1}{1 - 0.355 \text{ d } p/p_1} \quad d_p = p_2 - p_1$$

The exact value of γ plotted against γ_p in the graph, figure 3, illustrates equation (7). If an engine of $H\,P_m$ horsepower is available it follows that in order to obtain the increment of pressure p_2 desired for a quantity of air Q kg/sec the relation:

$$\Delta T_{me} = \frac{75 \text{ H } P_m}{E \cdot c_p \cdot Q} = 0.744 \frac{H\,P_m}{Q} \tag{8}$$

must be satisfied.

With the term of the mechanical compression phase, and with the assumption that the air leaves the compressor at the same speed v_r as at inflow, the compressed air finally assumes the effective temperature T_2:

$$T_2 = T_1 + \frac{\Delta T_m}{\gamma} \tag{9}$$

6. MINIMUM INTERNAL SPEED OF THE COMPRESSOR – MAXIMUM COMPRESSION

The residuary speed v_r can be computed in some cases while in others it cannot be achieved a priori. It must be put in ratio with the maximum section available in the interior before heating and with the total compression attained in this section Ω. If the residuary speed in this maximum section is reduced to v_m it follows that on exit from the compressor an effective increment of temperature

$$\Delta T_r = \frac{v_r^2 - v_m^2}{2 \text{ g } E \text{ } c_p} \tag{1}$$

is produced at the maximum section, as well as an effective dynamic pressure p_3/p_2 which, with θ as thermodynamic efficiency, affords, as in the first phase:

$$\frac{p_3}{p_2} = \left(\frac{T_2 + \Delta T_r}{T_2''}\right)^{\frac{\chi}{\chi-1}} \text{ with } T_2'' = T_2 + (1 - \theta)\, \Delta T_r \tag{2}$$

and

$$\frac{p_3}{p_0} = \frac{p_1}{p_0}\,\frac{p_2}{p_1}\,\frac{p_3}{p_2}$$

as ratio of compression in the maximum section Ω.

This ratio can be expressed in simple fashion by combining the final increment ΔT_r with the initial increment ΔT_d or by making the assumption that the initial dynamic compression of the outside speed v continues toward the speed v_m existing in the maximum section behind the compressor and in consequence neutralizes the ultimate compression $p_3 - p_2$. This assumption, which in practice produces no appreciable changes in temperature nor in pressure, simplifies the analytical formulas and permits sufficiently exact conclusions. In the following, therefore, the speed v_r (cf. equation 3, section 5) is replaced by the speed v_m relating to the maximum section Ω and the ratio Z, by introducing the pressures and temperatures, so that:

$$\boxed{Z = 1 - \frac{v_m^2}{v^2} = 1 - \left(\frac{\omega\, p_0\, T_2}{\Omega\, p_2\, T_0}\right)^2} \tag{3}$$

This formula affords the value of Z necessary to compute the dynamic compression. The compression phase p_3/p_2 is then included in the dynamic compression, whence the new residuary speed follows at

$$\boxed{v_m = v\,\frac{\omega\, p_0\, T_2}{\Omega\, p_2\, T_0}} \tag{4}$$

7. HEATING AND COMBUSTION

With C_m denoting the calories absorbed by the engine per kilogram of compressed air, and e the efficiency factor of the engine, the heat changed to energy W_m at the compressor shaft is only $e\ C_m$ while the remainder

$$C_r = C_m - e\ C_m$$

represents the heat not utilized and which in consequence is dissipated by the engine through the radiator, the engine itself and the exhaust gases. On the other hand the effective calories $e\ C_m$ produce, according to equation (4), for each kilogram of fluid the mechanical energy W_m (in calories) amounting to

$$\frac{W_m}{E} = e\ C_m = c_p \frac{\Delta T_m}{\gamma}$$

In addition the residuary calories C_r, when utilized for heating at constant pressure the same kilogram of air with specific heat C_p, produce a temperature rise in accordance with

$$\Delta T_r = \frac{C_r}{c_p} = \frac{(1 - e)\ C_m}{c_p}$$

Hence the total heat produced by the compressor assembly (the actual heating includes friction losses) in the assumption for disregarding the increment ϵ of the mass produced by the fuel amounts to:

$$\boxed{\Delta T_{me} = \frac{\Delta T_m}{\gamma} + \frac{1 - e}{e} \frac{\Delta T_m}{\gamma} = \frac{\Delta T_m}{e\gamma}} \qquad (1)$$

which, for the rest is evident and readily obtainable. By means of the action of the compressor unit the compressed air attains thus the temperature T_3 given by

$$T_3 = T_1 + \frac{\Delta T_m}{e\gamma}$$

At this point a more or less powerful combustion takes place, as stated in the beginning, which, for each kilogram of air again raises the temperature of the term ΔT_c absorbed in consequence of heat C_c with K denoting the ratio C_c/C_m of the heat absorbed by the combustion to that absorbed by the engine, the specific heat C_p of the gas can be once more maintained constant; the resulting increment ΔT_c equal to K changes that produced by the isolated compressor unit, that is:

$$\boxed{\Delta T_e = K \frac{\Delta T_m}{e \gamma} \qquad K = \frac{C_c}{C_m}} \tag{1}$$

The foregoing assumption regarding the value of c_p is verified in practice (reference 8, p. 55, reference 9, p. 197) since in the case in question the combustion of air is limited to a very small fraction of air and so reaches low terminal temperature, where neither the factors of dissociation of the mixture of CO^2 and CO nor the variations in temperature intervene to appreciably modify the values of c_p. The terminal temperature $T_4{}'$ from which the last phase of expansion is initiated is defined by:

$$T_4{}' = T_1 + \Delta T_{me} + \Delta T_c = T_1 + \frac{1 + K}{e\gamma} \Delta T_m \tag{2}$$

and at that in the absence of heat losses in respect to the outside of the propulsion unit. In fact, this situation closely resembles actual conditions, whether due to the low temperatures in play or because of an opportune heat insulation which is easily established on the outside surface of the unit. Still, in any attempt to prevent a loss it should be kept proportional to the difference $T_4 - T_o$ and so can be expressed by a factor which multiplies ΔT_d and ΔT_m in such a way that the temperature T_4 before discharge reads

$$T_4 = T_o + \varphi \Delta T d + \varphi \frac{1 + K}{e \gamma} \Delta T_m \tag{3}$$

with φ ranging between 0.98 and 1, according to the type of insulation employed but independent of the temperature jump $T_4 - T_o$.

8. INTERNAL PRESSURE LOSS DUE TO FRICTION AND HEATING

At the point where the compressed gas is heated to initial temperature T_4, its expansion obviously still exists up to the already computed pressure p_2 since given the low speed reached by the gas inside the propulsion system, the loss of pressure due to wall friction is insignificant. To illustrate: in case that the total compression p_2/p_0 is equal to 2.20 the specific volume of the fluid is approximately half of that of the ambient fluid. If, on these premises, the maximum inside diameter of the unit were equal to 1.8 times the diameter of the circle ω of the inlet, the inside area, in this instance, would be by about 3.20 greater than area ω and the internal velocity V_m would then be about 6.4 times lower than the foward speed. For an operating speed of say 250 miles per second, for example, it would afford an internal velocity of 39 miles per second, the load loss of which would be of the order (of magnitude) of some centimeter of water column and hence of an order of magnitude of 1/1000 of the total pressure $p_2 - p_0 = 1200$ centimeters.

However, with v_m denoting the speed of discharge of the compressor, the pressure loss in the short length of duct leading to the discharge, may be expressed by the equation:

$$\Delta p_a = - \xi \frac{v_m}{2} \rho_m \tag{1}$$

where ξ is the loss coefficient.

But more important are the load losses deriving from combustion and heating. These losses can be computed as follows:

If T_e is the temperature before heating and T_4 that after heating, it is readily seen that with maximum area Ω remaining available the speed increases up to a value v_c given by:

$$v_c = v_m \frac{T_4}{T_2} \frac{p_2 - \Delta p_e}{p_2} \simeq v_m \frac{T_4}{T_2} \tag{2}$$

On the other hand, application of the momentum equation to the fluid compressed in the cylindrical path bounded by sections equal to Ω where the speed v_c and v_m and the density ρ_e and ρ_m exist, affords:

$$\Omega \Delta p_e = \Omega v_m \rho_m (v_c - v_m) \text{ e } \frac{\rho_m}{\rho_c} = \frac{T_4}{T_2} \tag{3}$$

Then the above variations in velocity define the following pressure variations:

$$\Delta p_c = \rho_m v_o v_m - \rho_m v_m^2 = \rho_m v_m^2 \frac{T_4 - T_2}{T_2} \tag{4}$$

To compute the loss in effective energy caused by the reduction in pressure Δp_c, it is convenient to refer to the pressure that can be produced internally, when, after heating, the residuary speed is changed into pressure.

In the absence of pressure loss the fluid at residuary speed V_m and density ρ_c deriving from heating would have produced the increment of pressure

$$\Delta p_1 = \rho_c \frac{v_m^2}{2} = \rho_m \frac{v_m^2}{2} \frac{T_2}{T_4} \tag{5}$$

while the increment, obtainable at speed v_c which belongs to the case of pressure loss, is on the contrary:

$$\Delta p'_1 = \rho_c \frac{v_c^2}{2} = \rho_m \frac{v_m^2}{2} \frac{T_4}{T_2} \tag{6}$$

But in both cases the same temperature T'_4 is reached, that is to say, by equivalence of total heat dissipated in the fluid in both cases, with $T'_4 = T_4 + v_m^2/2 g c_p E$.

In the subsequent expansion the variation in kinetic energy follows thus from the final pressure differences between the two cases, or from the effective loss which may be termed the absolute load loss

$$\Delta p = p_2 + \Delta p_1 - (p_2 + \Delta p'_1)$$

which by having recourse to equations (5), (6), (4), becomes

$$\Delta p = \rho_m \left(\frac{v_m^2}{2} \frac{T_2}{T_4} + \frac{v_m^2}{2} \frac{T_4}{T_2} - v_m^2 \frac{T_4 - T_2}{T_2} \right)$$

or simply:

$$\Delta p = \rho_m \frac{v_m^2}{2} \frac{(T_4 - T_2)^2}{T_2 T_4} \qquad (7)$$

The loss in pressure due to friction and heat can be expressed by

$$\Delta p = \rho_m \frac{v_m^2}{2} \left(\xi + \frac{(T_4 - T_2)^2}{T_2 T_4} \right)$$

Bearing in mind equation (1) section 5 with $d R = 0$ which ties the small variations of kinetic energy to the small pressure variations in the adiabatic expansions it is concluded that the pressure variations produce the loss of kinetic energy in the discharge referred to one kilogram of fluid given by

$$\boxed{\Delta W = \frac{v_m^2}{2 g} \left(\xi + \frac{(T_4 - T_2)^2}{T_2 T_4} \right)} \qquad (7')$$

When $T_4 - T_2$ is lower than $T_2 T_4$, as occurs when the temperature prior to expansion is below $400° - 500°$C, the pressure losses by a given small value of ξ, can reach at the most the value corresponding to the dynamic pressure belonging to the residuary speed in the maximum section after double compression. In the contrary case the losses can be accurately defined by the foregoing formulas. But in a general study, as in this instance, allowance for the low internal pressure losses means to consider the total kinetic energy loss that corresponds to the residuary speed in the maximum section. This assumption is unfavorable at internal temperatures below $300° - 400°$, but beneficial at temperatures above it and may serve even as basis for a very accurate calculation when the maximum internal section has an area of not less than two or three times ω.

Following the appraisal of the pressure losses we pass to the ultimate phase of operation.

9. EXPANSION

The gas at pressure p_2 and temperature T_4 (previously computed), begins its expansion to pressure p_o and temperature T_s, producing a theoretical discharge velocity defined by

$$\frac{V^2}{2\ gEc_p} = T_4 - T_s = T_4 \left(1 - \frac{T_s}{T_4}\right) \tag{1}$$

But, according to the equation for the adiabatic expansion

$$\frac{T_s}{T_4} = \left(\frac{p_o}{p_2}\right)^{\frac{X-1}{X}}$$

where the ratio T_s/T_4 is identical with ratio T'_o/T'_2 of the adiabatic analogy of expansion which from temperature T'_2 and pressure p_2 reaches temperature T'_o and pressure p'_o. Bearing in mind that this is found in section 5,(6), and in section 6,(3).

$$T'_o = T_1 - \theta\ \Delta\ T_d \qquad T'_2 = T_o + \Delta\ T_d + \Delta\ T_m \quad T_1 + \Delta\ T_m$$

we easily find

$$1 - \frac{T_s}{T_4} = 1 - \frac{T_1 - \theta\ \Delta\ T_d}{T_1 + \Delta\ T_m} = \frac{\theta\ \Delta\ T_d + \Delta\ T_m}{T_o + \Delta\ T_d + \Delta\ T_m} = X \tag{2}$$

This ratio X defines the efficiency with which the calories subjected to constant pressure along the isobar p_2 are changed into kinetic energy of discharge. This derives from the fact that the factor

$$X = 1 - \frac{T_s}{T_4} = 1 - \left(\frac{p_o}{p_2}\right)^{\frac{X-1}{X}} \tag{2'}$$

represents remarkably close the efficiency of an ordinary cycle formed by any two adiabatics between the extreme pressures p_2 and p_o.

Putting for simplicity

$$\boxed{\frac{\Delta T_m}{\theta \Delta T_d} = r} \qquad \boxed{\frac{T_o + \Delta T_d}{\theta \Delta T_d} = \alpha} \tag{3}$$

the equation X of the efficiency of the fundamental internal cycle assumes the simple form

$$\boxed{X = \frac{1 + r}{\alpha + r}} \tag{4}$$

where r and α have an important physical significance. Term r in fact represents the ratio between the power of the mechanical and the dynamic compression and is a parameter of considerable influence at high speed on the value of the propulsive efficiency. This parameter r takes the place of the ratio between the two respective compressions, mechanical and dynamic, by simplifying the analytical expressions through elimination of the fractional exponent.

As concerns term α, it is readily seen that it represents the inverse of the efficiency of the fundamental cycle X, in absence of mechanical compression.

In this case, in fact, the cycle develops between pressure p_1 and p_o and assumes the value

$$X = 1 - \left(\frac{p_o}{p_1}\right)^{\frac{\chi - 1}{\chi}}$$

which, as is seen, coincides with the formula

$$X = 1 - \frac{T'_o}{T_1} = \frac{\theta \Delta T_d}{T_o + \Delta T_d} = \frac{1}{\alpha}$$

A comparison of equation (3) above with the expression for ΔT_d in section 5, (3) affords

$$\boxed{\alpha = \frac{1980\ T_o + z\ v^2}{z\ \theta\ v^2}} \tag{5}$$

which enables finding α by given operating speed and ambient temperature T_{os}, notable being the experimental coefficients z and θ.

Entering the term X in equation (1) the rate of discharge V is then evolved from

$$\frac{V^2}{2\ g\ E\ c_p} = \psi\ X\ T_4 \tag{6}$$

where ψ allows for the friction losses in the discharge duct. The above equation can be written also for equation (3), section 8.

$$\frac{V^2}{2\ g\ E\ c_p} = \psi\ X \left(T_o + \varphi\ \Delta\ T_d + \varphi\ \frac{1 + k}{e\gamma}\ \Delta\ T_m\right)$$

which, making use of the second above, can be expressed in the form

$$\frac{V^2}{2\ g\ E\ c_p} = \psi\ 6\ \Delta\ T_d + \psi\ \Delta\ T_m - \psi\ X\ (1 - \varphi)\ \Delta\ T_d$$

$$+ \psi\ X\ \frac{\varphi\ (1 + k) - e\gamma}{e\ \gamma}\ \Delta\ T_m \tag{7}$$

10. KINETIC ENERGY AND EFFICIENCY

With the rate of discharge V obtained for evaluating the kinetic energy it is necessary to refer to the equation (4), section 1, but for securing a simple efficiency formula, it is recommended to compute the increment of kinetic energy of discharge produced by compression and heating with respect to that obtainable with the single dynamic compression.

In this last case for sections 5 and 7 in absence of combustion or mechanical compression, and residuary speed discounted, the air compressed by the single dynamic compression is discharged at rate v' according to

$$\frac{v'^2}{2\ g\ E\ c_p} = \psi\ \theta\ \Delta\ T_d \qquad \boxed{v'^2 = \psi\ z\ \theta\ v^2}$$

Inversely the kinetic energy at discharge in the case of the preceding paragraph, bearing in mind that 1 kilogram of air is mixed with ϵ kilogram of fuel, is:

$$\alpha\ \frac{V^2}{2\ g\ E\ c_p} = \frac{W_s}{E\ c_p} \qquad \alpha = 1 + \epsilon$$

Hence the portion of the kinetic energy actually produced by the action of the propulsion system and the combustion follows at

$$W' = W_s - \frac{v'^2}{2\ g} = E\ c_p\ \psi \left[\alpha\ \Delta\ T_m + \alpha X\ \frac{\varphi\ (1 + k) - e\gamma}{e\ \gamma} + (\alpha - 1)\ \theta\ \Delta\ T_d - (1 - \varphi)\ X\ \alpha\ \Delta\ T_d\right]$$

But in order to produce the temperature increment induced between compressor, heating and combustion given by $\frac{1 + k}{e\ \gamma}\ T_m$ for the weight of air α with mean specific heat c_p the total calories

$$C_t = \alpha\ \frac{1 + k}{e\ \gamma}\ \Delta\ T_m\ c_p$$

are needed.

From this the efficiency produced by the kinetic energy follows at

$$\frac{W'}{E\ C_t} = \psi\ B = \psi\ \frac{e\ \gamma + X\ [\varphi(1 + k) - e\ \gamma]}{e\ \gamma} + t$$

Bearing in mind that the term

$$t = \frac{e\gamma\ \Delta\ T_d\ \psi}{d(1 + k)\ \Delta\ T_m} \left[(\alpha - 1)\ \theta - (1 - \varphi)\ \alpha\ X\right]$$

which largely represents the effect of the mass increase and of the cooling losses is zero for $\alpha = 1$ and $\varphi = 1$, that is, practically zero for internal temperatures below 300° to 400°, and since the difference of the two quantities is generally quite small, it is practically always justified to ignore them in relation to the other terms, hence write

$$\boxed{B = \frac{e\,\gamma + [\varphi(1 + k) - e\,\gamma]\,X}{1 + k}} \qquad (1)$$

without introducing appreciable errors.

Comparing this equation with (4) and (5) in section 1, while bearing in mind that obviously

$$W = \frac{\alpha\,V^2 - v^2}{\alpha\,V^2 - v'^2}\,W' \quad \text{and putting} \quad \frac{V}{v} = R \qquad (2)$$

we find

$$\boxed{y = m\,u\,\psi\,B} \quad \text{with} \quad \boxed{u = \frac{\alpha\,R^2 - 1}{\alpha\,R^2 - z\,\theta\,\psi}} \qquad (3)$$

where all terms can be expressed as functions of parameters α and r and of the previously introduced coefficients.

The ratio $R^2 = \frac{V^2}{v^2}$ is given by equation (6) in the preceding paragraph and equation (2) of section 5, at

$$R^2 = \frac{\psi\,X\,T_4\,z}{\Delta\,T_d}$$

and is readily secured by introducing the above parameters along with equation (7), section 7.

$$R^2 = \psi\,z\left[\theta\,(1 + r) + X\,\frac{\varphi\,(1 + k) - e\,\gamma}{e\,\gamma}\,r\,\theta - X\,(1-\varphi)\right]$$

From this formula, with the fact in mind that $1-\varphi$ is quite small compared to the two others preceding it and that its omission would not produce any appreciable changes in the ratio R and also that such elimination giving a ratio R greater than actual always entails a small variation in less of term m, the simplified formula follows at:

$$\boxed{R^2 = \psi z \theta \left[1 + r + r X \frac{\varphi (1 + k) - e \gamma}{e \gamma}\right]} \tag{4}$$

With R^2 and hence R we immediately compute

$$\eta = \psi m u = \psi \frac{2 (\alpha R - 1)}{\alpha R^2 - z \psi \theta} \tag{5}$$

and the propulsive efficiency

$$y = \eta B \tag{6}$$

whence the consumption in grams per horsepower per hour given by equation (3) section 6 reads:

$$\pi = \frac{57.50}{y} \text{ g/hp/hr}$$

With these formulas we can readily secure the value of the propulsive efficiency in relation to the two fundamental parameters α and r that define the operation of the propulsion system, and plot the curves for the different operating conditions.

When K = 0, or in the absence of combustion, equation (1) affords:

$$B = e \gamma + X (\varphi - e \gamma) \tag{7}$$

When r = 0, or in the absence of mechanical compression, equation (1) section (8). ($K = \infty$ and hence according to equation (1) and (6) section 7 and equation (4) section 10) affords:

$$B = \varphi X \quad y = \eta \varphi X = \eta \frac{\varphi}{\alpha} \tag{8}$$

When $a = \infty$ or for equation (5) section 10, at very low speed, in the absence of combustion and at low compression, hence when r is not very high,

$$B \simeq e\gamma \qquad y \doteq \eta \gamma e \tag{9}$$

may be retained.

From equation (6) it is established that the term

$$\boxed{\eta = \varphi m u}$$

given by (5) represents solely the propulsive efficiency comprising all losses due to friction and to kinetic energy of discharge, a term that exactly corresponds to the propeller efficiency.

Term B instead represents exlusively a thermal efficiency and can take the place of the thermal efficiency of the engine-propeller system, while observing that term B is directly dependent upon the efficiency η and cannot be examined separate from it, except in the special case where we can retain $X = 0$ and hence $B = e\gamma$ or else $K = \infty$ and hence B = constant.

The efficiency of propulsion alone η follows as function of R only between the speed of discharge and the speed of translation since the factors φ, z and θ can be kept constant. The curve of this efficiency is shown in the chart, figure 4, for

$$\alpha = 1.01, \quad \psi = 0.99 \quad z = 0.97, \quad \theta = 0.96$$

which resembles average experimental values.

It generally has a maximum obtainable by derivation from equation (5) equating to zero and resolving the corresponding equation. The maximum for R_m follows from

$$R_m = 1 + \sqrt{1 - \psi \alpha z \theta} \tag{10}$$

in accord with which the efficiency assumes the simple form

$$\boxed{\eta_{max} = \frac{\psi \alpha}{R_m}} \tag{11}$$

In the specific case where B is constant or independent of R this maximum coincides with the maximum propulsive efficiency y which defines the speed of advance. But in a general case, the maximum propulsive efficiency y being variable with R, y occurs at a value of R which more or less differs from that of R_m, whence the necessity to plot the curves defining each operating speed by points. It is obvious then that, once the value of r and R have been secured, even a substantial variation of these values from that defining the maximum will produce practically no change in the propulsive efficiency.

12. EXPERIMENTAL COEFFICIENTS

The values of the various coefficients can be determined experimentally.

As concerns the term ϵp, Aymerito's test data (reference 11) on jets of 16 millimeter diameter in the contracted section, or those by Castagna at Reynolds numbers of around 5.10^5 or the more recent data by Ackeret (reference 10, and 24) can be used.

In these experiments the ratio of absolute to theoretical compression was defined. By deducing from this the ratio ϵ_p between the excess pressures, starting from the pressure in the minimum section, a mean pressure efficiency of 0.90 to 0.92 is afforded for speeds below 1100 meters per hour. Aymerito's value was 0.95. (In both these test series the Reynolds number effect was determined.)

In general the pressure efficiency obtained with the large exit cones of wind tunnels is less (reference 12).

Tests in the Caproni tunnel, for instance, gave scarcely 0.75 in spite of the benefit of large Reynolds numbers. This is a direct result of the deflection of the air after passing through a chamber of immobile air or at least a long cylindrical path of the jet where the boundary layer thickness is greatly increased, and so becomes largely responsible for the separation from the walls of the cone as soon as a minimum recompression is effected.

(This is shown clearly in Tomaso's experiments who found efficiencies of 0.98 - 0.99 in the absence of the boundary layer (reference 24).

When this boundary layer is minimum at the mouth of the exit cone as in the cited experiments and in the somewhat older tests reported by Andre's (reference 13) the efficiency also amounts to 0.90 - 0.92 for small jets (case of liquids).

In the case dealt with here there is no pre-existing boundary layer in the throat of the exit cone, hence the results are better, even for small diameters as borne out by experiment.

Important systematic studies carried out in collaboration with the Caproni Company have also brought out the extremely great influence of the form of the jet and of the Reynolds number. It may be stated that for large jets (Reynolds numbers of $5-10\times10^6$ and speeds < 1000 km/h) a pressure efficiency of 0.96 can be easily obtained and a resultant thermodynamic efficiency of around 0.96 - 0.98 at subsonic speeds, that is, up to about 1100 kilometers per hour. (A report on the efficiency at supersonic speeds is to follow in the near future.)

The pressure efficiency γ_p depends upon the type of compressor used. A high-power centrifugal compressor, designed by modern standards, as regards blade sections and fillets, can give an efficiency of $\gamma_p = 0.85$. An axial compressor with few discontinuities, and of the same diameter as those used in the modern large wind tunnels, guarantees an efficiency even higher and a quasi-adiabatic compression. Experiments made in the Caproni laboratories with an axial compressor (sections derived from the NACA airfoil No. 0020 with thickness and camber varying progressively along the radius (reference 23)); having a 460 millimeter impeller afforded an efficiency of about 0.85, which agrees very closely with that obtainable on the basis of blade profile drag at the Reynolds number of the test (at blade half about 3×10^5), so that on full-scale axial impellers (Reynolds number about 2×10^6) a practical thermodynamic efficiency of from 85 to 90 percent or perhaps even higher (reference 20) could be counted on.

These results are not unusual when it is considered that a propeller gives a propulsive efficiency which may be as high as 0.85, hence a blower efficiency of the order of 0.90 to 0.92 notwithstanding the losses by wake eddies, losses which are recovered to a great extent in axial compressors by the diffuser or straightener that neutralizes the free axial vortex.

The efficiency of discharge ψ is notably high and closely approaches unity for convergent jets and subsonic speeds. At speed superior to the effects of thrust reaction, a partial, rather than a complete, entrance cone (effusor) is sufficient to improve the efficiency of the discharge cone that obtains with a complete Laval nozzle (reference 1, Part II) and which is known to be 0.95 to 0.96 for small diameters (references 14, 8, 15).

Tests at small Reynolds numbers (maximum about 10^6) made in the Caproni laboratories with jets of 300 - 400 millimeters diameters at speeds of the order of 50 meters per second gave an efficiency of around 99 percent. For large jets at large Reynolds number ($Ry = b \div 10 \times 10^6$) this efficiency is higher than 99 percent at any discharge velocity (below 500 - 600 meters per second for gas at 300°). Recent tests by the NACA on jet propulsion (reference 5) with respect to the coefficient ψ, disclosed an efficiency consistently above 99 percent on the basis of a theoretical thrust reaction of $\varphi = 1$ and an effective thrust reaction for jets with diameters of a few inches and speeds up to 300 - 400 meters per second.

Of the remaining coefficients: $\alpha = 1 + \epsilon$ the mass increase and φ the heat loss, the first is a function of the increment of the total temperature and can be computed as follows: if P is the lower heat value of the fuel and C_p the specific heat of the mixture of air and combustion products, the increment of the temperature ΔT for each kilogram of air is:

$$(1 + \epsilon)\, c_p\, \Delta T = \epsilon P \qquad \boxed{1 + \epsilon = \frac{P}{P - c_p \Delta T} = \alpha} \qquad (1)$$

where ϵ is the amount of fuel per kilogram of air.

The increment ΔT, beyond the dynamic compression, is, in accordance with the foregoing arguments,

$$\Delta T = \frac{(1 + k)}{e\gamma} \Delta T_m$$

which, bearing in mind the relations (3) section 10, readily affords

$$\boxed{\Delta T = \frac{(1 + k)}{e\gamma} \frac{\theta r}{\theta a - 1} T_o} \qquad (2)$$

To illustrate: for values ΔT of the order of 500° and with gasoline P, 11,000 calories, we get

$$1 + \epsilon = \frac{11000}{11000 - 120} = 1011$$

The coefficient of heat loss φ cannot be evaluated by disregarding the type of thermic insulation adopted in practice. But, to take as example the most unfavorable case, that is, without insulation with an exposed area of 10 m^2 and a temperature jump of 500° between inside and outside, assuming a coefficient of transmission by convection and radiation in air at density 1/3 of that in the ground of Ke = 40, the heat loss per second is:

$$\Delta C = 40 \times 10 \times 500^\circ/3600'' = 55 \text{ cal/sec}$$

Since such a propulsive unit would have the dimensions corresponding to the development of horsepower at altitude of about 250 horsepower, or 439 calories per second the resultant consumption with y = 0.20 would be about 2200 calories per second. Hence, the loss in this case would be reduced to 2.2 percent without insulation. With thermic insulation this loss can, of course, be reduced to insignificant values, that is, values of less than 1/100 of the consumption.

13. ILLUSTRATIVE CALCULATION OF THE PROPULSIVE EFFICIENCY

(a) Operation without combustion - speed 500 kilometers per hour.

Let us compute the efficiency of a propulsion system capable of operating at 500 kilometers per hour at 500 meters without the aid of combustion. In this case a coefficient of heat loss of $\varphi = 0.99$ may be assumed.

We get v = 139 meters per second. Suppose, on the other hand, an

Adiabatic efficiency of diffuser	θ = 0.96
Coefficient of maximum internal heat loss.	z = 0.97
Efficiency of discharge cone	φ = 0.99
Adiabatic efficiency of compressor........	γ = 0.87
Augmentation of mass......................	α = 1.005
Efficiency factor of engine	e = 0.24

Then by apportioning a ratio r = 1 for mechanical and dynamic compression, and allowing for the fact that the temperature at standard air at 5000 meters is $-17^\circ = 256^\circ$ K, we obtain according to formula 5, section 10:

$$a = \frac{1980 \times 256^\circ + 0.97 \times \overline{139^2}}{0.97 \times 0.96 \times \overline{139^2}} = 29.30$$

whence, immediately follows, according to equation (4) section 10:

$$X = \frac{1 + 1}{29.30 + 1} = 0.066$$

as the efficiency of the adiabatic cycle. In the absence of combustion, that is, with K = 0 the thermal efficiency of the group becomes, according to equation (7) section 11:

$$B = 0.24 \times 0.87 + 0.066 \, (0.99 - 0.24 \times 0.87) = 0.260$$

To find the mechanical efficiency it is necessary to secure the speed of discharge or the ratio R. According to equation (4) section 11 it is:

$$R^2 = 0.99 \times 0.97 \times 0.96 \left[1 + 1 + 0.066 \, \frac{0.99 - 0.24 \times 0.87}{0.24 \times 0.87} \right] = 2.072$$

hence R = 1.440 and for equation (5) section 11

$$\eta = 0.99 \, \frac{2 \, (1.005 \times 1.44 - 1)}{1.005 \times \overline{1.44^2} - 0.99 \times 0.97 \times 0.96} = 0.761$$

and finally

$$\gamma = 0.761 \times 0.260 = 0.198$$

Bearing in mind that, by assumption the efficiency factor of the engine is $e = 0.24$ it follows that in order to obtain an identical efficiency with the same engine but with a propeller, an identical efficiency, the propeller efficiency must be

$$\eta_e = \frac{0.198}{0.24} = 0.825$$

Modifying the efficiency factor 10 ÷ 20 percent more or less would produce an identical result about the equivalent propeller for 500 kilometers per hour speed of advance.

(b) Operation with combustion - speed 950 kilometers per hour.

Let us assume any altitude up to around 14000 meters where it is desired to realize a propulsion of the system at a speed of 950 kilometers per hour with the setting as in the preceding example, hence with the same resistance to advance, (discounting for the moment the effect of the compressibility on the value of the resistance itself). Then a new effective power

$$HP_{950} = HP_{500} \times \frac{950}{500} = 1.90\ HP_{500}$$

is necessary. Hence a 90 percent power increase must be provided.

Supposing, subject to check later on, that this power increase can be realized with a better efficiency than in the preceding case; that is $y = 0.22$. In this case the consumption at 950 kilometers per hour would be higher than that at 500 kilometers per hour according to the relation

$$C_{950} = C_{500} \times 1.90\ \frac{0.198}{0.220} = 1.71\ C_{500} \text{ hence } K = 0.71$$

Repeating then the calculation with $K = 0.71$ and $\alpha = 1.01$ while retaining the same coefficients and bearing in mind that the new altitude will be attained at 14000 meters approximately where the standard air temperature is 56°, 5 C°, and hence $T_0 = 216°, 50$ we find:

$$a = \frac{1980 \times 216°,50 + 0.97 \times \overline{264^2}}{0.97 \times 0.96 \times \overline{264^2}} = 7.65$$

Assuming further in this case $r = 1.00$ we get

$$X = \frac{1 + 1}{7.65 + 1} = 0.2313$$

whence the thermal efficiency follows according to equation (1) section 11 at

$$B = \frac{0.87 \times 0.24 + 0.2313\ (0.99 \times 1.71 - 0.87 \times 0.24)}{1.71} = 0.323$$

The speed of discharge is obtained as before at

$$R^2 = 0.99 \times 0.97 \times 0.96\left(1 + 1 + 0.2313\ \frac{0.99 \times 1.71 - 0.87 \times 0.24}{0.87 \times 0.24}\right) = 3.36$$

whence follows:

$R = 1.832$ and with $\alpha = 1.01$

$$\eta = 0.99\ \frac{2(1.01 \times 1.832 - 1)}{1.01 \times \overline{1.832^2} - 0.99 \times 0.97 \times 0.96} = 0.680$$

$$y = \eta\ B = 0.22$$

This efficiency could also be obtained with an engine-propeller system having an engine with a horsepower 71 percent greater than that necessary for a speed of 500 kilometers per hour but with a propeller efficiency of

$$\eta = \frac{0.22}{0.24} = 0.916$$

It further follows that the consumption $K = 0.71$ produces, in effect, the propulsive power increment

$$\frac{N'}{N} = 1.71 \frac{0.22}{0.198} \doteq 1.90$$

with respect to the preceding case.

(c) Case of engine stoppage - speed 950 kilometers per hour.

Assume case (b) in which the engine stops while running at 950 kilometers per hour. The efficiency in this instance would become

$$y = \eta \frac{\varphi}{a}$$

as seen in section 11.

The thermal efficiency B thus assumes a constant value:

$$B = \frac{\varphi}{a} = \frac{0.99}{7.65} = 0.1225$$

and the propulsive efficiency with $\psi = 0.99$ $\alpha = 1.01$ becomes

$$y = 0.1225 \times 0.99 \frac{(1.01\ R - 1)\ 2}{1.01\ R^2 - 0.931}$$

Under these conditions the efficiency is then a sole function of the chosen ratio R of speed of inflow and of discharge. The maximum efficiency can be obtained from equation (11) section 11, in which

$$R_m = 1 + \sqrt{1 - 0.931} = 1.2625$$

It obviously is obtainable for values of R ranging between R = (1.20 - 1.50) since, being located at the point of maximum on curve y = f (R) figure 4, a variation in R does not change the value of y and assumes the value

$$y_{max} = \frac{0.1225}{1.2625} = 0.1025$$

So, in order to obtain the propulsive horsepower necessary for flight in case the engine stops, the consumption increases in the ratio

$$\frac{C'}{C} = \frac{0.22}{0.1025} = 2.15$$

over that of the previous case.

Hence the propulsive power required can equally be obtained, when the engine stops, but the consumption then increases by 111 percent at 950 kilometers per hour speed; at lower speeds it would increase in much greater proportion.

(d) Operation without combustion - V = 380 kilometers per hour.

Leaving the setting of the assembly unchanged its speed at zero altitude would become about 380 kilometers per hour. Under these conditions an atmospheric temperature $t + 15^\circ$ that is, $T = 288^\circ$ always with $r = 1$, - affords

$$v = 105.60 \qquad a = \frac{1980 \times 288 + 0.97 \times \overline{105.60^2}}{0.931 \times \overline{105.60^2}} = 56.60$$

hence

$$X = \frac{1 + 1}{56.60 + 1} = 0.0347$$

and the thermal efficiency becomes, with $K = 0$:

$$B = 0.24 \times 0.87 + 0.0347 \; (0.99 - 0.24 \times 0.87) = 0.2364$$

The ratio R^2 is given in the usual manner by

$$R^2 = 0.922\left(2 + 0.0347\,\frac{0.791}{0.209}\right) = 1.965$$

hence

$$R = 1.401 \qquad \eta = 0.99\,\frac{(1.005 \times 1.401 - 1)2}{1.005 \times 1.965 - 0.922} = 0.767$$

and finally

$$y = 0.767 \times 0.2364 = 0.1815$$

According to this it would be possible to obtain at zero level the same result with the same engine and with an equivalent propeller capable of an efficiency of

$$\eta_e = \frac{0.1815}{0.24} = 0.76$$

Hence at low speed, in the absence of combustion, the equivalence with the engine propeller unit is complete.

14. OPERATING CHARTS

The calculations of the foregoing examples were repeated with $\gamma = 0.835$ for a large number of values of parameter r and in a way so as to secure an efficiency curve for each speed and altitude.

In addition, the successive values 1, 2, 4, 8, 16, 32 and ∞ were given to factor $K + 1$ for each chosen speed and plotted against parameter r for each propulsive efficiency chart.

The base speeds were assumed at altitudes corresponding approximately to those obtainable with conventional equipment which at sea level, in the same setting as at high altitude, had a speed of around 380 kilometers per hour.* The variations in the efficiency resulting from an altitude other than that of the chosen basic altitude are very small and readily computed as will be shown below.

The results of this systematic study have been collected in charts I to V which refer to the cases in the following table:

Chart No.	Speed km/hr	Altitude (m)	Temperature °C
1	300	0	+ 15
2	500	5,000	- 17
3	700	10,000	- 50.5
4	900	14,000	- 56.5
5	1100	16,000	- 56.5

The six curves corresponding to values $1 + K$ are indicated in each chart by successive numbers. The curves corresponding to progressive consumption are referred to the engine consumption in the ratio $1 + K = c_t/c_m$.

In the absence of the compressor the propulsive efficiency attains, as previously noted, a value which corresponds to $K = \infty$. So instead of a curve, a point with abscissa $r = \theta$ defines this condition on the chart.

These charts require no further explanation. It is readily seen that with increased speed of advance the effect of factor r is slight at small combustions $K < 4$ but becomes profound for combustions absorbing from 4 to 30 times the consumption of the engine alone. The maximum of this curve shifts rapidly toward minimum compression and to zero for $K = \infty$. This ties in with the fact that

$$R^2 = \psi z \theta \left[1 + r + r \times \frac{\varphi(1 + K) - e\gamma}{e\gamma} \right]$$

* The altitudes are included for the sole purpose of establishing their order of magnitude. Temperature and speed must be fixed. The time altitude can be accurately secured when the aircraft characteristics are known.

with ascending K, that is, the internal temperature, increases rapidly and this rise is so much faster as X is higher or as the speed is greater, and in consequence of which the kinetic or propulsive efficiency m drops quickly to very low values unless low ratios of compression or minimum values of r are used.

If the speeds plotted in each chart had been obtained at an altitude other than that shown in the tabulation the corresponding curves would manifest slight variations. But instead of varying the curves it is sufficient to assign to each chart a speed diagram different from the basic speed, since the curves themselves correspond exactly to the different situation.

In fact, all the other coefficients being equal, the term a is the only one that varies with the speed, hence identical curves correspond to two situations which have an identical value of term a. According to the definition for a, identical curves correspond to two situations defined by speed v and v_1, and temperature T_o and T_{o1} if the relation

$$a = \frac{1980\ T_o + z\ v^2}{z\ \theta\ v^2} = \frac{1980\ T_{o1} + z\ v_1^2}{z\ \theta\ v_1^2}$$

is satisfied. From it the new speed v_1 corresponding to the temperature T_{o1} differing from that assumed for computing the above chart follows at

$$v_1^2 = \frac{1980\ T_{o1}}{z(a\ \theta - 1)}$$

where a is the value which corresponds to the chart in question and hence

$$v_1 = v\sqrt{\frac{T_{o1}}{T_o}} \tag{1}$$

The foregoing relation enables the direct deduction of the new speeds corresponding to each chart for temperatures other than those on which the plot is based.

The variations are, however, minimum at high altitude as seen from a study of the temperatures at altitude, while at low levels they would be considerable and other than negligible.

All the plots are separated by the horizontal $y = 0.19$ which corresponds to the efficiency of an engine-propeller unit with a propeller efficiency of $\eta_e = 0.80$ and with an engine identical with that mounted on the jet propulsion system.

It is noted that in absence of combustion the efficiency of the jet propulsion unit, hence its range is consistently greater than that of the engine-propeller unit at speeds above around 450 kilometers per hour and slightly below at lower speeds. (This result refers to the case $\gamma = 0.835$; with $\gamma = 0.87$ the equivalence shifts toward 380 - 400 kilometers per hour). At a speed of 1000 kilometers per hour the efficiency with $K = 0$ would be 0.29, hence the range, in relation to an equivalent engine-propeller unit, increased by about 47 percent while at 300 kilometers per hour the efficiency would be 0.176 and the range about 7 percent less. Equivalence is reached at about 380 - 400 kilometers per hour.

This last situation with $K = 0$, when compared with the boosting of air in the engine which, as seen very shortly can be pushed to 2 - 3 atmospheres, can be of interest for long range aircraft capable of very high speed.

The rise in efficiency in this instance compared to an equivalent engine-propeller system is largely the result of the utilization of heat dissipated by the engine. It was found that the combustion always produces a drop in efficiency, the percentage being so much more accentuated at low speeds and less at high speeds; so, for example, at 1100 kilometers per hour with $1 + K = 4$, that is, with an engine developing only 25 percent of its total power, an optimum efficiency is still obtained.

Since, in general, the absence of combustion calls for high engine horsepower and hence a lower useful load by equal gross weight of the aircraft, the greater range at higher speed is always obtained with a more or less high consumption to be defined for each case according to the aircraft characteristics. Such study is facilitated by the effective power diagrams which are given at the end of the present report, following the analysis of the internal temperatures and pressures of the propulsion system.

15. MAXIMUM INTERNAL PRESSURE AND TEMPERATURE

The operating diagrams must be completed with a systematic examination of the total pressure and maximum temperature, so as to permit an accurate examination of the eventual reduction in engine horsepower, or to study the limits of applicability of the charts in relation to the maximum temperatures obtained in correspondence with the different values of r or of the various mechanical compressions. Following this the diagrams, enabling the plotting of the efficiency curves at various height levels in relation to the maximum speed obtained without combustion, are illustrated.

(a) The compressions produced in correspondence to a given value of r and a follow from equation (2) section 10, where, with $X = 1.41$, a value corresponding to the case of air with $t \leq 200$ to 300°, we get

$$\boxed{\frac{p_z}{p_o} = \frac{1}{(1 - X)^{3.44}}} \tag{1}$$

Since X is a function of r, p_z/p_o also is a function of v, so the curves of total compression and of mechanical compression can be plotted for each speed.

To illustrate: at a speed of 950 kilometers per hour it affords with $r = 1$

$$\frac{p_z}{p_o} = \left(\frac{1}{1 - 0.2305}\right)^{3.44} = 2.466$$

while for $v = 0$ (engine stopped)

$$\frac{p_z}{p_o} = \left(\frac{1}{1 - 0.1303}\right)^{3.44} = 1.62$$

hence $r = 1$ corresponds to a total compression of 2.46 and a mechanical compression of $2.46/1.62 = 1.52$. By this method the curves of the ratios of total compression and mechanical compression have been plotted against r in each diagram for each speed. These curves, put in relation with the relative density at the effective height where the speed of the diagrams were obtained, enable a quick orientation in the choice of a more appropriate value of r in the different cases for canceling or at least minimizing the drop in engine* horsepower at altitude which is expected in every case with the usual high altitude compressor

(b) The maximum temperature reached inside the propulsion system is given in equation (3) section 8, which, starting from atmospheric temperature T_o, gives the total increase:

$$T'_4 - T_o = T_o \left(\frac{\Delta\ T_d}{T_o} + \frac{1 + K}{e\ \gamma}\ \frac{\Delta\ T_m}{T_o}\right)$$

whence, with

$$\frac{T_o + \Delta\ T_d}{\theta\ \Delta\ T_d} = a \qquad \frac{\Delta\ T_m}{\theta\ \Delta_d} = r$$

in mind, follows

$$\boxed{T'_4 - T_o = T_o\ \frac{1 + \theta\ r\ (1 + K)/e\ \gamma}{\theta\ a - 1}} \qquad (2)$$

For the speed and altitude of exmaple (b) the above equation becomes, for instance:

$$T'_4 - T_o = 216.50\ \frac{1 + 4.55\ r\ (1 + k)}{6.15}$$

for $r = 1$ and $K = 0.71$ it is

$$T'_4 - T_o = 35.2^o + 160^o\ r\ (1 + K) = 309.2^o$$

* The dynamic and mechanical compression work in series beyond the effect of the compressor.

whence the temperature in °C in the example is

$$T_{max} = 309.2^\circ - 56.5^\circ = 252.7^\circ$$

Plots Ia to Va show this maximum internal temperature plotted against r for different values of 1 + K. Case r = 0 cannot be shown in the temperature charts. In this instance the increment $T'_4 - T_o$ is expressed as function of R. In fact, according to equation (4) section 11, with $K = \infty$; $r = 0$, we get

$$R^2 = z \psi \theta \left(1 + \frac{\varphi \nu K}{a e \gamma}\right)$$

while the foregoing equation (2) affords

$$T'_4 - T_o = T_o \frac{1 + \theta \nu K/e \gamma}{a \theta - 1}$$

with $v = 0$; $K = \infty$; and, after elemination of $r k/e \gamma$:

$$\boxed{T'_4 - T_o = \frac{T_o}{\theta a - 1}\left(1 + \frac{a (R^2 - \psi z \theta)}{\psi z \varphi}\right)} \qquad (3)$$

So, with R = 1.40, it affords in example (c)

$$T'_4 - T_o = 35.2^\circ \left(1 + \frac{7.65 (\overline{1.40^2} - 0.922)}{0.99 \times 0.97 \times 0.99}\right) = 295^\circ$$

and the internal temperature

$$t_{max} = 295 - 56 = 239$$

if, with engine cut out, the dimensions of the propulsion system enable it to realize R = 1.40

(c) It is also necessary to know in function of r, the aspect of the temperature immediately following the two successive stages of dynamic and mechanical compression and whether for checking the value of Z resulting from the effective density after the compression or for examining the cooling capacity of the compressed air which leaves the radiator and feeds the engine. This temperature is quickly secured from the foregoing relation by putting $K = 0$ and $e = 1$ as is readily seen from a comparison with equation (9) section 6.

Thus the increment produced by the dynamic and the mechanical compression alone follows at

$$\boxed{T_2 - T_0 = T_0 \frac{1 + \frac{\theta r}{\gamma}}{\theta a - 1}} \qquad (3)$$

In the cited example this increment is given by

$$T_2 - T_0 = 35.2^\circ (1 + 1.09) = 73.6^\circ$$

whence the temperature t_2 after compression is:

$$t_2 = 73.6^\circ - 56.5 = + 17.10^\circ$$

Now, the velocity v_m in the section Ω which evidently satisfies

$$v_m = v \frac{p_0}{p_2} \frac{T_2}{T_0} \frac{w}{\Omega}$$

can be accurately computed.

The ratio

$$\boxed{\frac{\rho_0}{\rho_2} = \frac{p_0}{p_2} \frac{T_2}{T_0} = f\ (r)} \qquad (4)$$

which is also included in the graphs enables a quick calculation of v_m/v for a given design ratio ω/Ω and hence of coefficient z.

Lastly it is useful to know the power required for the mechnanical compression of 1 kilogram of air in correspondence to each value of r.

This specific power in horsepower per kilogram on the basis of 1 calorie per second = 5.70 horsepower and hence 1^o per second X kilogram = 1.345 horsepower is simply expressed by

$$\boxed{\frac{HP}{Q} = 1.345 \frac{\Delta\ T_m}{\gamma} = 1.345 \frac{\theta\ r}{\gamma\ (\theta a - 1)}} \qquad (5)$$

where Q is the quantity of air (in kilogram) compressible per horsepower.

16. OPERATION OF COMPRESSOR AT CONSTANT POWER

The knowledge of the specific power of compression in function of r and of the quantity Q actually inductible by the inlet opening under the various conditions of flight enables the evaluation of the characteristic curve of the compressor in relation to chosen values of r at various speeds of advance, or the curve expressing the ratio of effective compression in function of Q, and vice versa, the prediction of the values of r at different speeds and heights for a given type of compressor.

The quantity inductible by the compressor in the various phases depends upon the design characteristics of the unit and may vary for each speed between fairly wide limits of the geometrical quantity that can be inducted on the basis of the opening of the inlet and the air density. The variations can be effected by varying the discharge parts (reference 1) so as to produce a positive or negative pressure in the actual throat of the intake ω_1 and hence a speed of inflow v_1 variable on the throat, while the speed v of the unit is maintained. The fundamental quantity itself is readily ascertained.

With $g\rho$ as specific weight of gas and ω_1 the real opening of the inlet, the fundamental quantity Q follows at

$$Q = g\ \rho\ w_1\ v \qquad (1)$$

This fundamental quantity varies considerably with altitude and speed by modifying the characteristic curve necessary for the compressor in the various cases of flight. Two types of fundamental operation or limiting cases are involved, namely:

(a) flight at various heights and speed with constant angle of setting,
(b) flight at constant altitude with speed and setting variable.

In case (a) with fixed fundamental setting of aircraft characterized by, say, setting for maximum fineness, or least energy and for a specified wing loading, the speed of advance is tied to the density through the relation

$$v = v_o \sqrt{\frac{\rho_o}{\rho}} \tag{2}$$

where v_o is the speed reached by the aircraft at zero altitude and density ρ_o in the setting under consideration. (This relation is in practice sufficiently approximate up to 900 to 1000 kilometers per hour. For higher speeds a certain approximation obtains for symmetrical profiles, while for cambered profiles every validity is lost (references 16, 17). In consequence law 2) can also express flight beyond 900 to 1000 kilometers per hour but with approximate changes in setting).

With Q_o as the quantity at zero altitude the fundamental quantity that can be inducted at various speeds is

$$Q = Q_o \frac{v_o}{v} \text{ kg/sec} \tag{3}$$

Hence the quantity (in kilogram) in flight at constant angle of setting decreases proportionally to the increase in the speed of translation. This result ties in with the formula for the mechanical compression attainable with an available horsepower and enables the deduction of value r, for the fundamental quantity, at constant horsepower and variable altitude.

With a_o denoting the value of a at speed v_o and zero altitude, equation 5, section 15 affords, with r_o as value of r at zero altitude,

$$r_o = \frac{HP_o}{Q_o} \frac{\theta a - 1}{1.345 \frac{\theta}{\gamma}}$$

for the basic quantity Q_o.

But with

$$\theta \alpha - 1 = \frac{T_o}{\Delta T_d} = \frac{1980 T_o}{z v^2}$$

taken into account (equation 3, sections 10 and 5) it readily follows that the ratio between the value of r at speed v and the value r_o is simply expressed by

$$\boxed{\nu = \nu_o \frac{v_o}{v} \frac{T}{T_o}} \qquad (4)$$

or by equal power of mechanical compression the ratio r between mechanical and dynamic compression for the geometrical quantity decrease proportionally to the rise in speed of translation and to the drop in absolute temperature. With an arbitrary value of r established the successive or preceding values at different speeds (always in flight at constant setting) can be immediately identified accurately on the speed diagrams, with the added convenience of adopting for each speed a slightly different value from the basic one, so as to increase, for example, the compression of gases feeding the engine.

Referring to the basic values of r as deduced by the foregoing method we get

$$r = \frac{\Delta T_m}{\theta \Delta T_d} = \frac{\Delta T_m 1980}{z \theta v^2}$$

hence

$$\frac{r}{r_o} = \frac{\Delta T_m}{\Delta T_{mo}} \frac{{v_o}^2}{v^2}$$

a formula which, put in ratio with the preceding one, gives

$$\boxed{\frac{\Delta T_m}{\Delta T_{mo}} = \frac{v}{v_o} \frac{T}{T_o}} \qquad (5)$$

Since the increments ΔT_m are proportional to the energy W_m imparted by the compressor to each kilogram of fluid, this specific energy of compression increases in proportion to the speed of advance.

Following this the volumetric quantity is put in ratio with the above energy W_m.

But with Ω as the intake of the compressor, v_r the speed in the said section and w_i the intake of the propulsion unit, it affords

$$v_\nu = v \frac{w_i}{\Omega} \frac{\rho_o}{\rho_1}$$

with

$$\boxed{\frac{T_1}{T_o} = 1 + \frac{\Delta T_d}{T_o} = \frac{\theta a}{\theta a - 1}} \quad \boxed{\frac{p_o}{p_1} = \left(\frac{a - 1}{a}\right)^{3.44}} \qquad (7)$$

derive from equation (3) section 10 and equation (6) section 5 the volumetric quantity of air per mc/sec.

$$Q_v = v_\nu \Omega = v w_i \frac{\theta a}{\theta a - 1} \left(\frac{a - 1}{a}\right)^{3.44} = v w_i F \qquad (7)$$

Hence, given the quantity Q_{vo} which corresponds to speed v_o and which can be obtained by the above formula for $a = a_o$, the different volumetric quantities can be secured from the formula

$$\boxed{Q_v = Q_{vo} \frac{v}{v_o} \frac{F}{F_o}; \qquad F = \frac{\theta a}{\theta a - 1} \frac{(a - 1)^{3.44}}{a^{3.44}}} \qquad (8)$$

where F and F_0 are the reduction factors resulting from the dynamic compression which are obtained from previous equations.*

To illustrate: the factor F in the assumption

$$\theta = 0.96 \qquad z = 0.97$$

assumes the aspect of that in figure 5 with respect to different speeds of advance and temperature, which results from a speed of 380 kilometers per hour at zero altitude. The factor changes, of course, when the speed at zero altitude is modified.

Case b, flight at constant altitude, being secondary in importance by reason of the high fuel consumption (reference 19) is discussed later.

18. CHARACTERISTIC CURVES OF COMPRESSOR WITH CONSTANT POWER

In the case of centrifugal or axial compressors with fixed blade angles, with N_0 denoting the rpm at zero level and V_0 the speed, since at similar speeds the energy imparted to the fluid is proportional to the square of the rpm, it follows that, being able to change the rpm, it can be made to satisfy equation (5) section 16 very simply by means of a change in N_0 so that

$$\sqrt{\frac{\Delta T_m}{\Delta T_{mo}}} = \frac{N}{N_o} = \sqrt{\frac{v}{v_o}\frac{T}{T_o}} = n$$

that is, by shifting the operation of N_o (figure 6) which expresses the energy ΔT_m in relation to the volumetric quantity $Q_v = \Omega v_v$, toward curve N which is obtainable from the preceding curve (reference 20, p. 176, reference 21, p. 210).

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*This factor is none other than the relative densities of the outside atmospheric conditions.

However, no matter what operation is maintained in strict analogy, by passing from curve N_0 to N, the speed v_v in the inlet orifice must vary in proportion to the ratio N/N_0; and vice-versa, having defined the effective variation of v_v in relation to the cause previously examined, the given points cannot correspond to the affined ones, but rather are characterized by the new volumetric quantity Q_v obtainable from equation (8) and energy ΔT_m deriving from equation (5).

Although appreciable, the displacements have no material effect on the efficiency and by whatever method the foregoing quantity is modified in respect to the basic quantity, we can, if necessary, plot the operation to the most convenient one, or to the speed in strict affinity.

To illustrate, with an assumed basic speed of $v_0 = 385$ kilometers per hour, the successive speeds follow at:

km/hr	T_0	$\frac{\Delta T_m}{\Delta T_{mo}}$	$\sqrt{\frac{\Delta T_m}{\Delta T_{mo}}} = n$
385	288	1.00	1
500	256	1.15	1.072
700	233	1.41	1.187
900	216.50	1.76	1.326
1100	216.50	2.15	1.467

On the other hand, regarding the speed in the intake in relation to the geometrical quantity the variations are as follows:

km/hr	p_1/p_0	T_0/T_1	F	v/v_0	$\frac{Q_v}{Q_{vo}} = n'$	n'/n*
$385 = v_0$	1.066	0.990	0.950	1	1	1
500	1.128	0.965	0.920	1.30	1.256	1.17
700	1.309	0.920	0.830	1.81	1.590	1.34
900	1.558	0.873	0.735	2.35	1.810	1.36
1100	1.900	0.823	0.640	2.85	1.925	1.32

*(This ratio represents the distance of removal of the affinite speed).

In this case, fixing the maximum efficiency of the compressor at $n'/n = 1.17$, for instance, at speeds above or below 500 kilometers per hour the quantity could be varied up to 20 percent from that giving maximum efficiency, hence the compressor would always operate at maximum efficiency. Figure 6 illustrates this behavior of the compressor at different forward speeds and heights. (Instead of plotting the prevalence-quantity curves for different rpm we plotted those for $\sqrt{\text{prevalence}}$ - quantity, where the affinite points are aligned with the origin, rather than on the parabola).

A quasi-identical result can be obtained by a variation of the rpm, a variation which requires a change in speed, with an appropriate determination of the characteristic compression curve, or by other mechanical means customary in the manufacture of compressed air. In this respect there are any number of possible solutions according to either the characteristics of the heat engine or of the employed compressor.

But for the purposes at hand it is sufficient to show the operational variations of the compressor required at different speeds in flight with constant setting at high altitude.

This example shows that the variations n in rpm will be minimum in comparison to the speed changes of the aircraft (while the speed changes from 380 to 1100 kilometers per hour or 180 percent, the rpm rise from 1 to 1.467 or scarcely 46 percent).

These small variations can be annulled or minimized by proper care, as stated before, so as to approximately insure constancy in engine and compressor in flight at different speeds and rpm.

Contrariwise, in flight at constant altitude, $\rho_o T_o$ is constant, hence

$$\boxed{\frac{\Delta T_m}{\Delta T_{mo}} = \frac{v_o}{v} ; \quad \frac{Q_v}{Q_{vo}} = \frac{v}{v_o}} \qquad (9)$$

or any rise in speed is accompanied by a rise in the volumetric quantity and a drop in the energy ΔT_m, hence, a farther removal from the the basic point which is defined by the maximum efficiency.

But since the increases in speed afforded at constant height are much inferior to those of the preceding case, satisfactory operation is readily attainable in this instance, also, either by utilization of the elasticity of the compressor, which, if properly designed, allows for ample changes in quantity while retaining high efficiency, or else by changing the effective quantity in respect to the basic quantity as previously outlined.

Figure 7 represents the efficiency and the experimental power of an axial compressor especially designed to absorb a constant horsepower and to furnish a constant efficiency for wide variations in volumetric quantity. The tests agree, as shown, with theory.

As regards the variations of r in flight at constant altitude, they are readily secured by combining the previous equation with the formula for r given in equation (3):

$$\frac{r}{r_o} = \frac{Q_o}{Q} \frac{v_o^2}{v^2} = \frac{v_o^3}{v^3} \tag{10}$$

Thus r decreases in flight at constant altitude very rapidly with the square of the forward speed. This decrease at increasing speed maintains the propulsive efficiency near its maximum value, by reason of the fact that at constant altitude the increments of the power require strong combustions, hence large values of $1 + K$ and, on the other hand, as $1 + K$ increases the maximum efficiency is, as a matter of fact, removed toward the minimum values of r as manifested by the propulsive efficiency curve. On passing from one speed to a higher one, the new curve has a K value which can be obtained from the relation

$$1 + k_v = \frac{y_o}{y} \left(\frac{v}{v_o}\right)^3$$

on the assumption that the horsepower required for flight rises with the square of the speed, where y and y_o are the efficiencies involved in the two situations.

Thus a brief examination of the behavior of the compressor in flight at different altitudes and speeds with engine at constant power brings us to the study of the behavior of the unit at variable altitude and speeds with engine of decreasing power, — a case that is inevitably presented when the altitude is increased beyond a certain limit.

This study is held in abeyance pending its ultimate development at a future date in a manner fitting its importance and in connection with new experiments under way.

For the present it suffices to mention their existence and significance for the accurate prediction of the behavior of the system at high altitude and various speeds.

Translation by J. Vanier,
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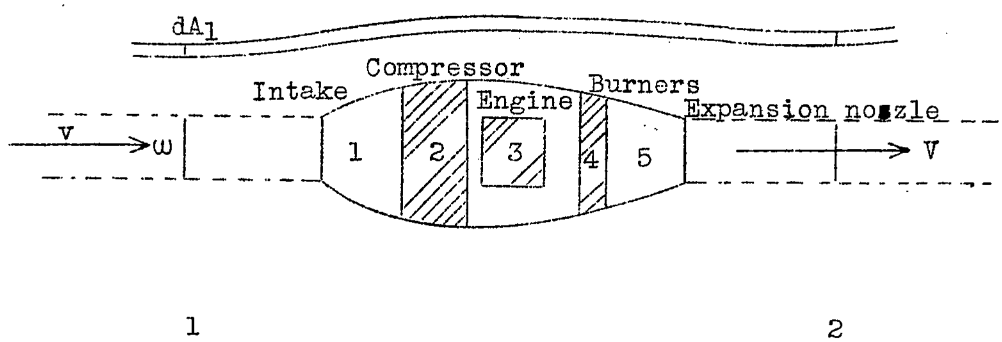


Figure 1.-

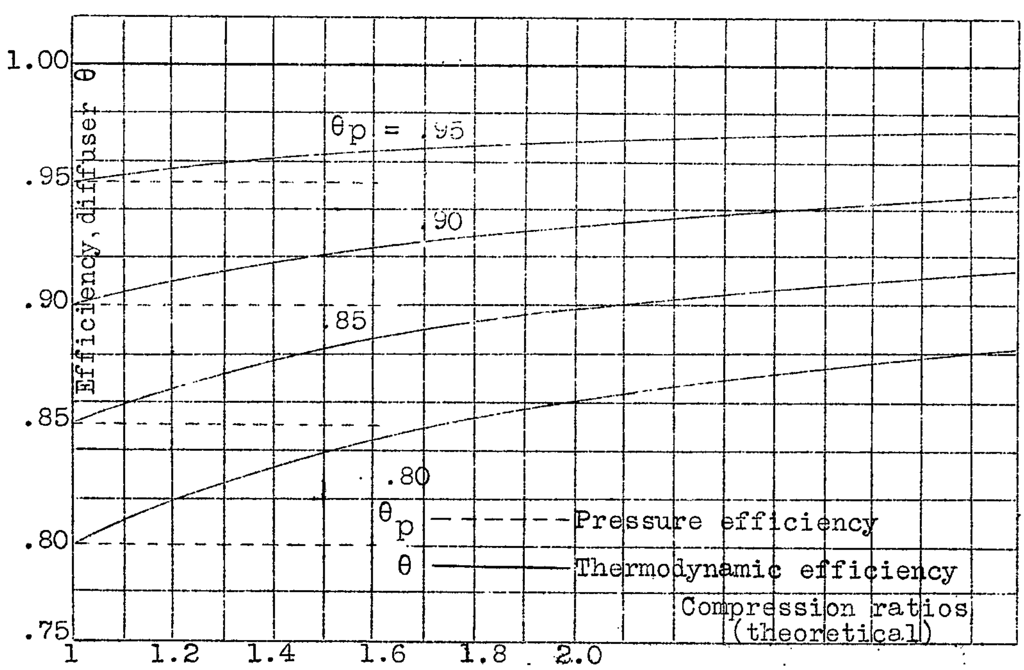


Figure 2.- Efficiency of diffuser.

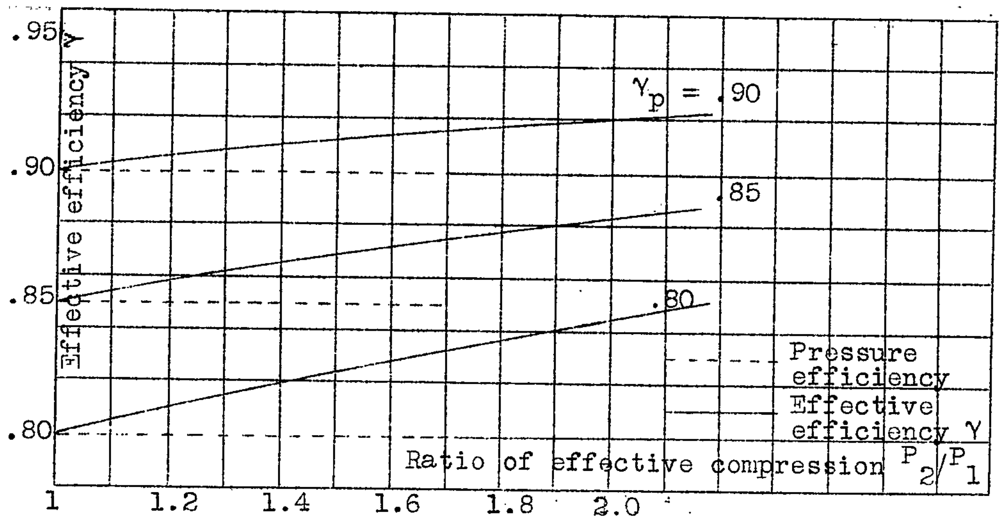


Figure 3.- Efficiency of compressor.

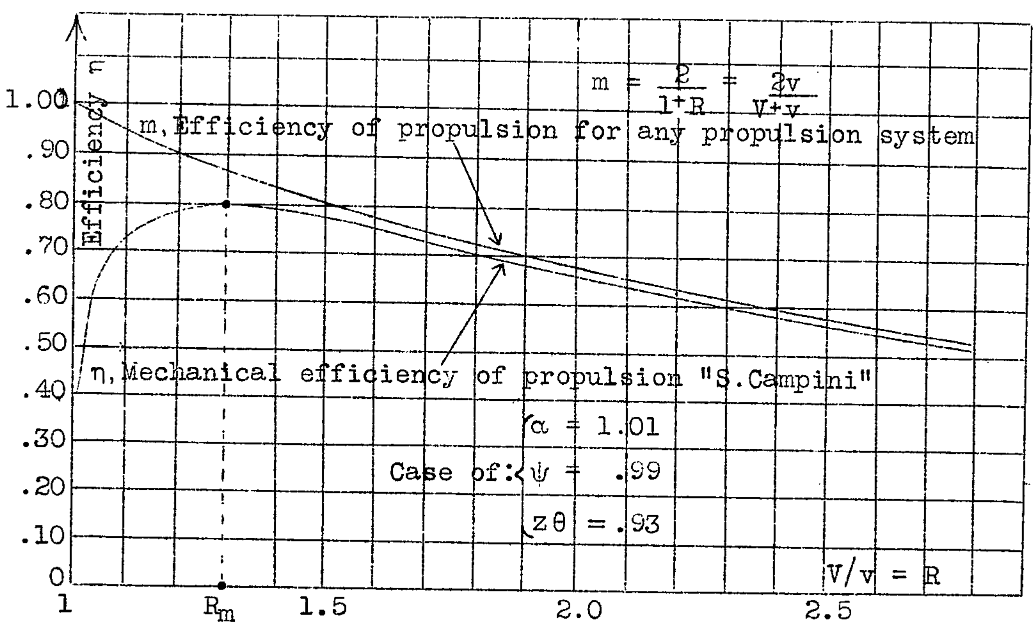


Figure 4.- Efficiency of propulsion. Mechanical efficiency η of propulsion, Campini.

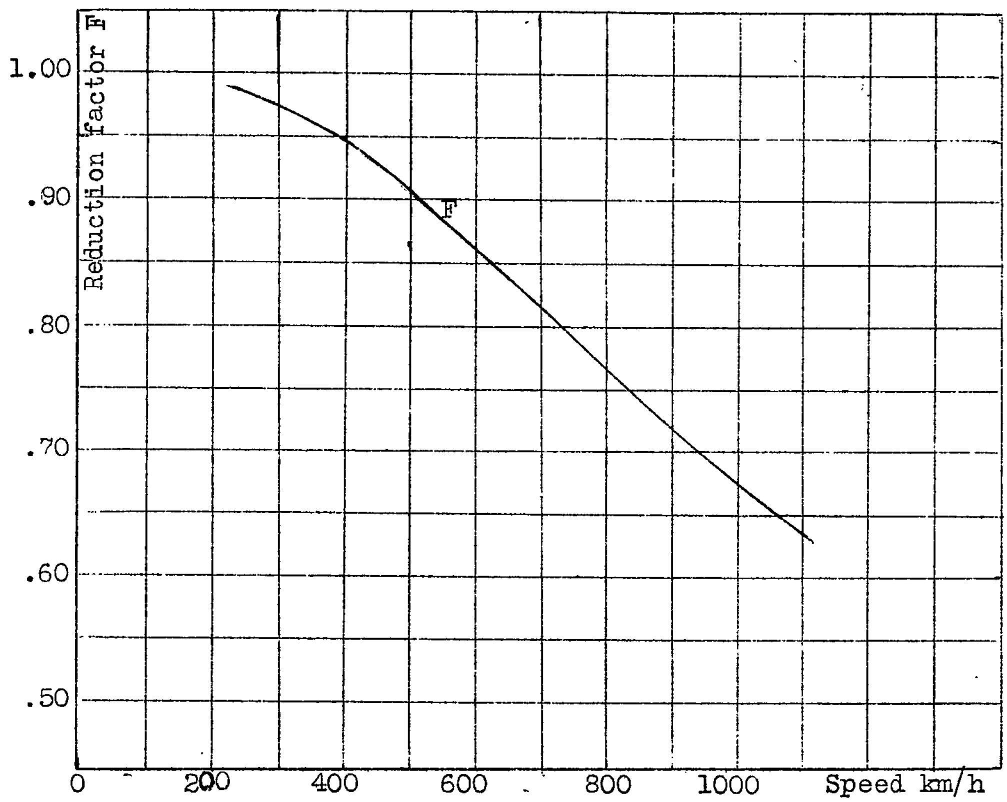


Figure 5.- Relative density 1:F. Factor F.

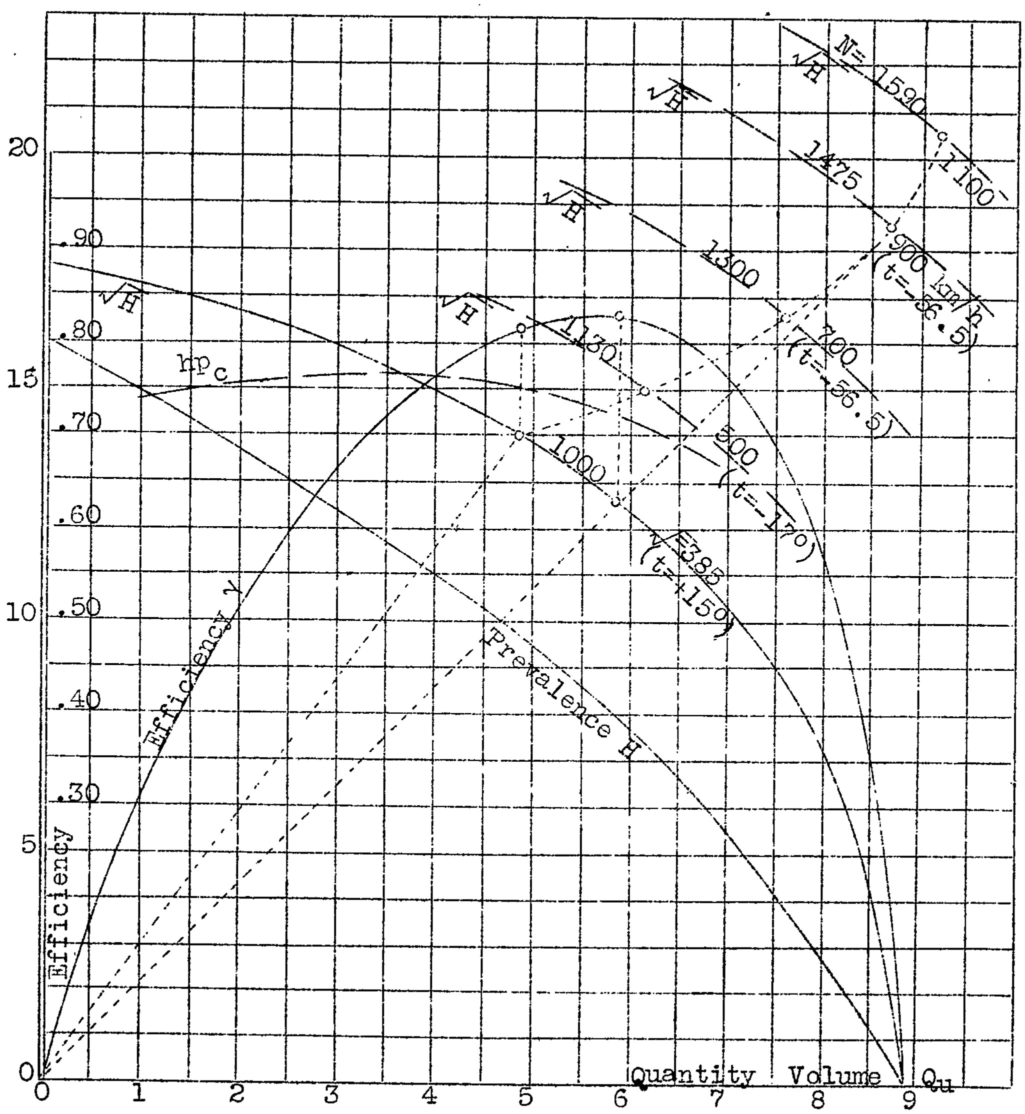


Figure 6.- Compressor curves.

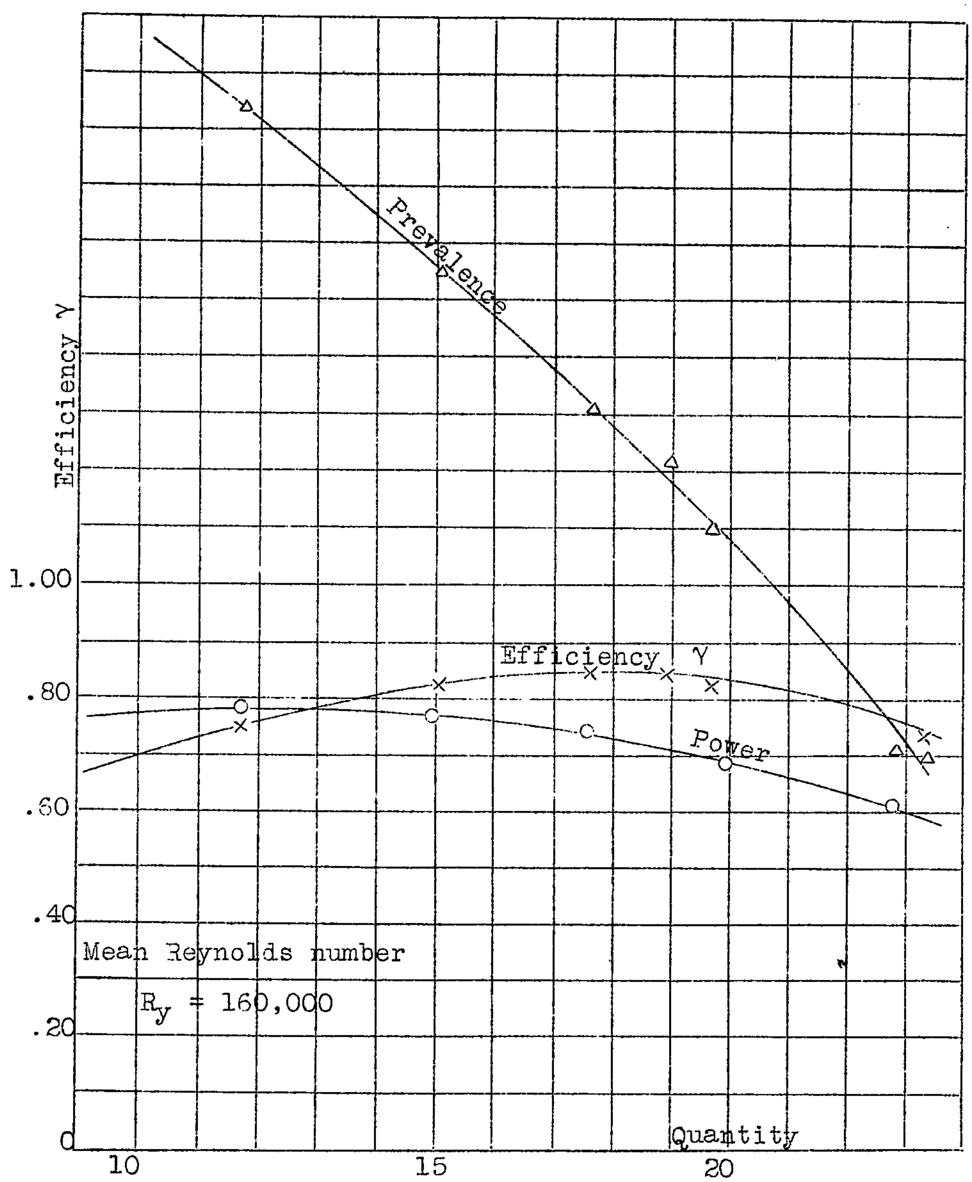
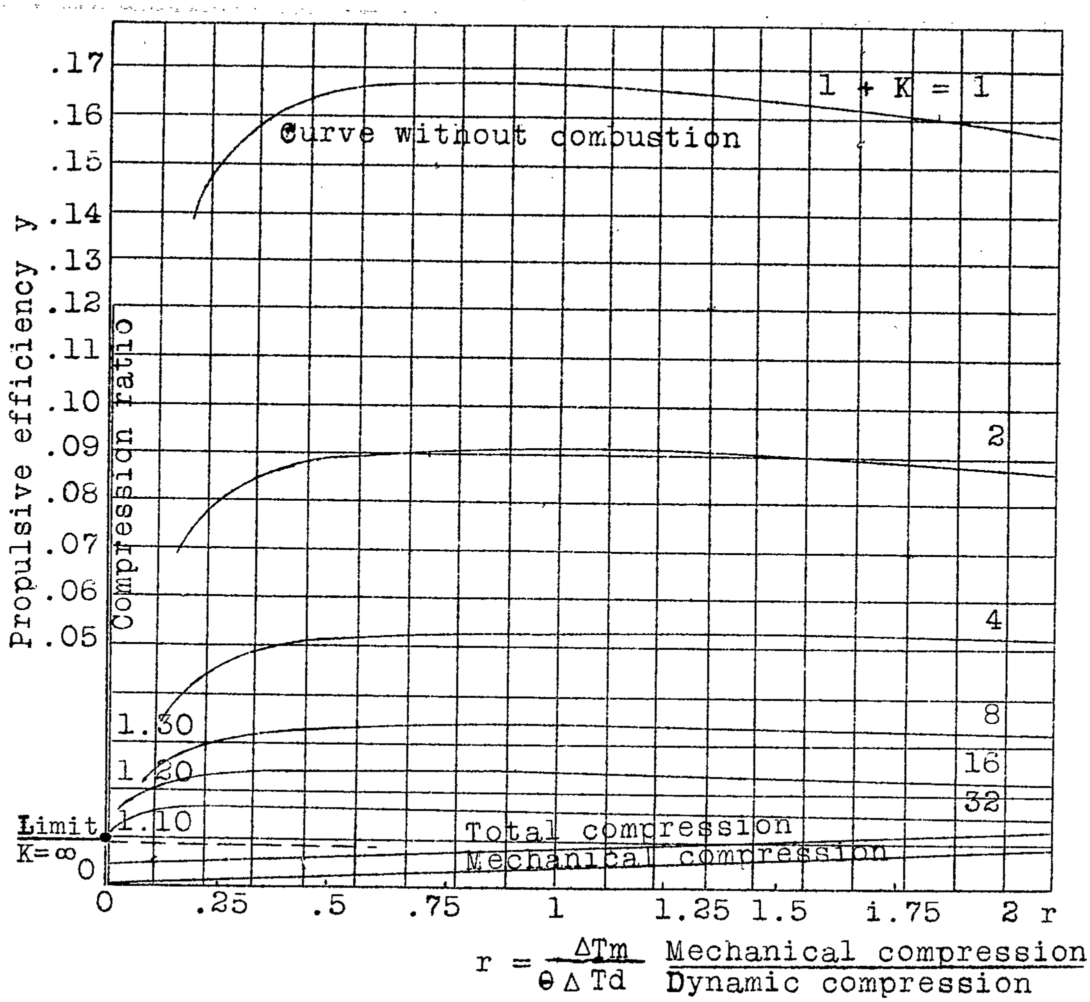
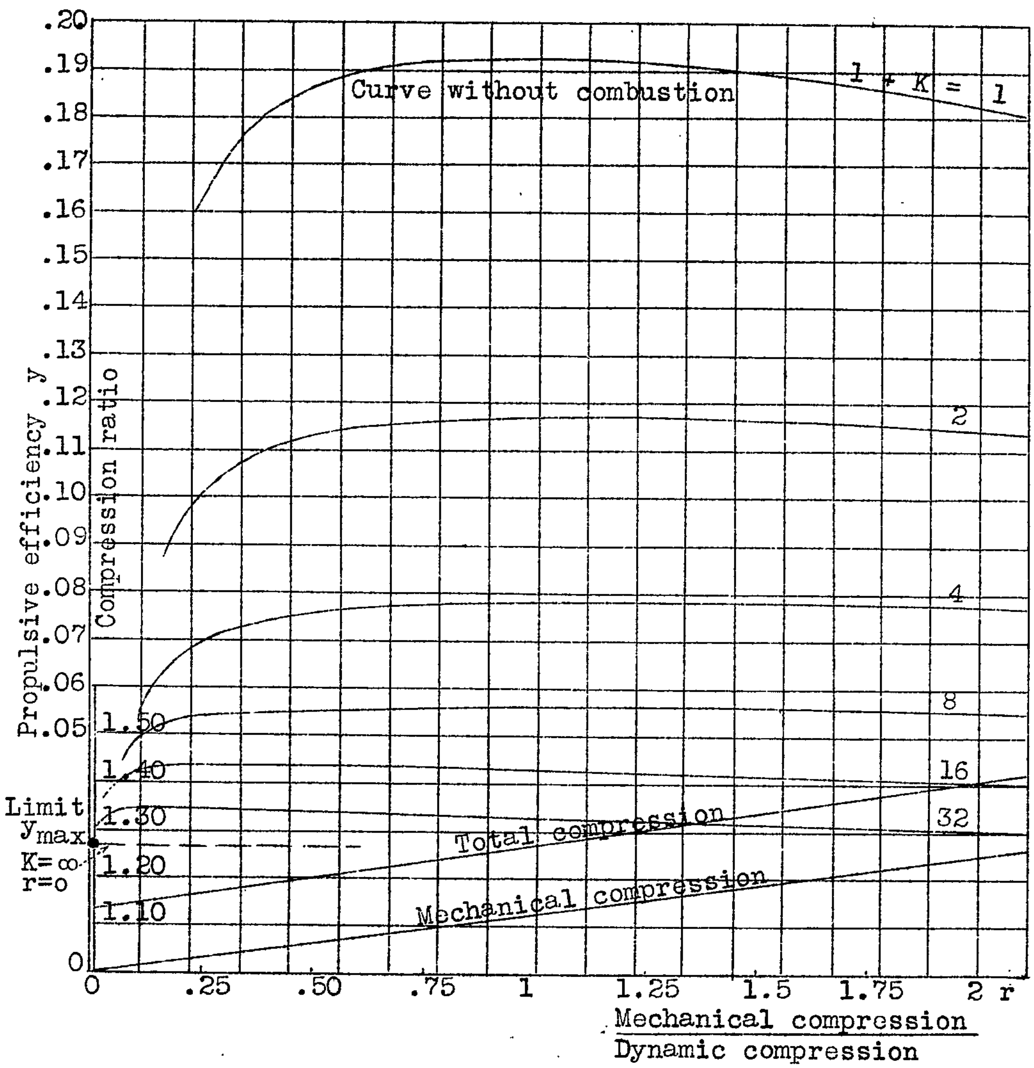


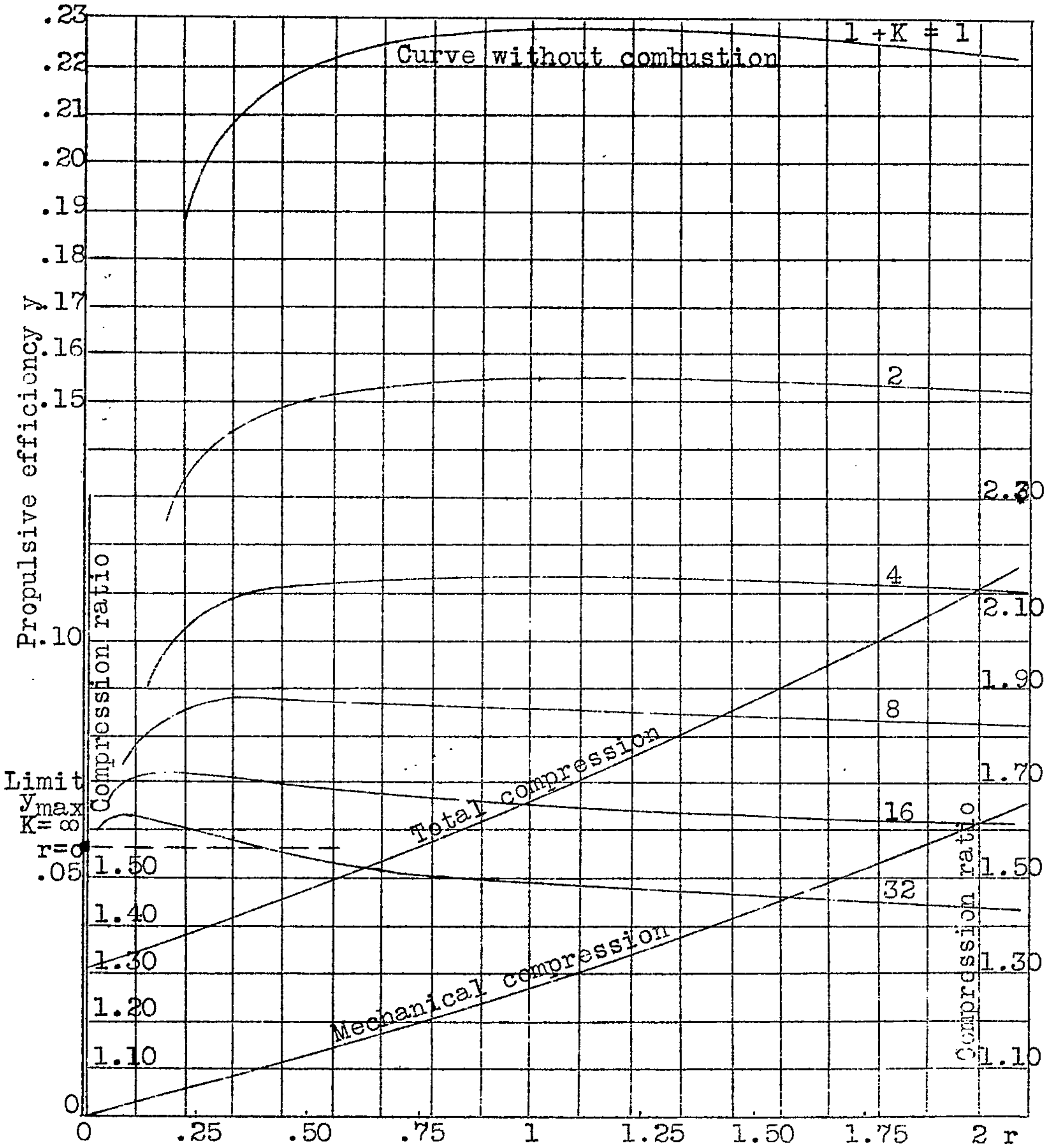
Figure 7.- Axial compressor. Experimental curves.



Plot 1.- Temperature.
v = 300 km/h
t = + 15° C.

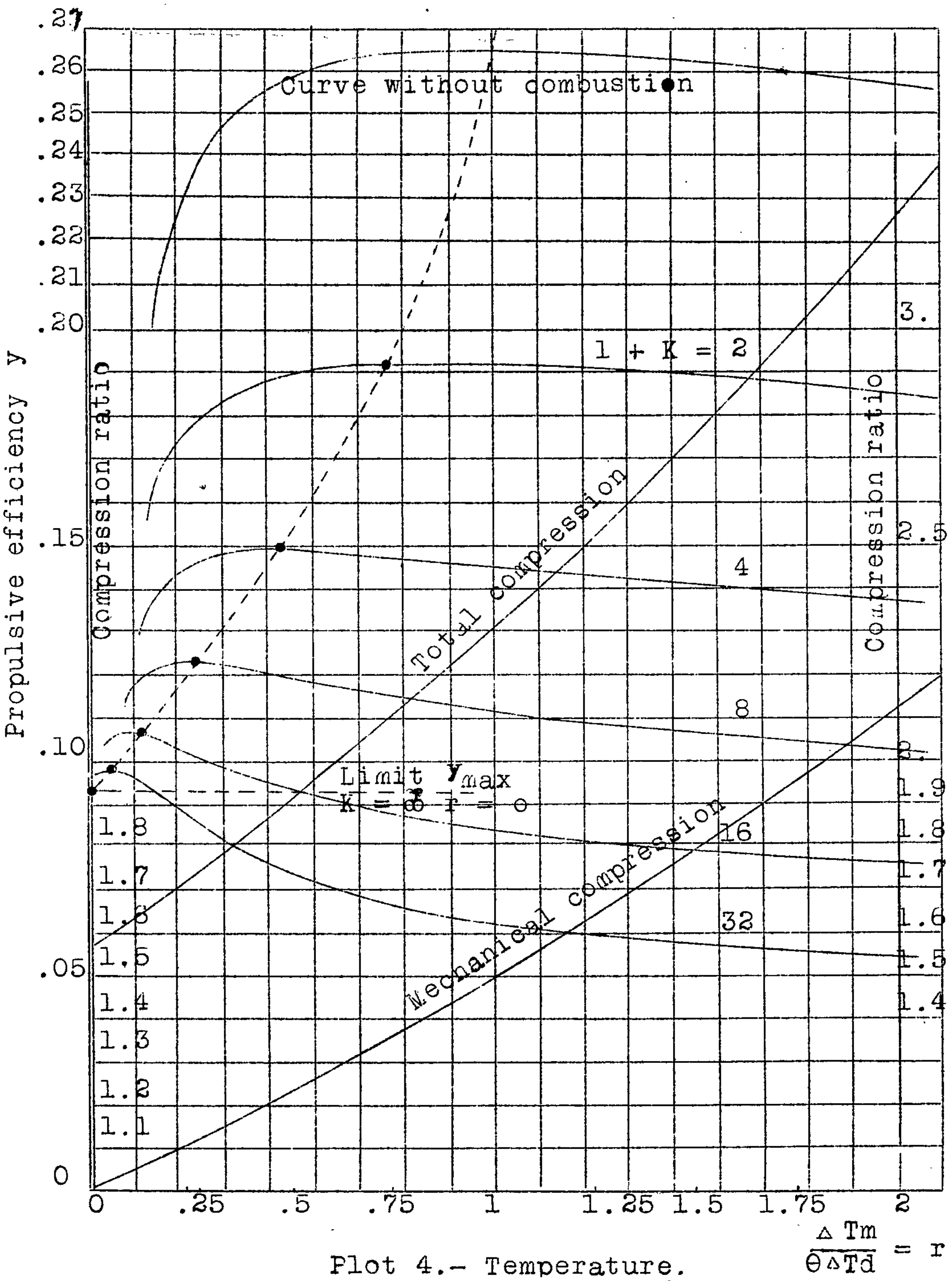


Plot 2.- Temperature.
v = 500 km/h
t = - 17° C.

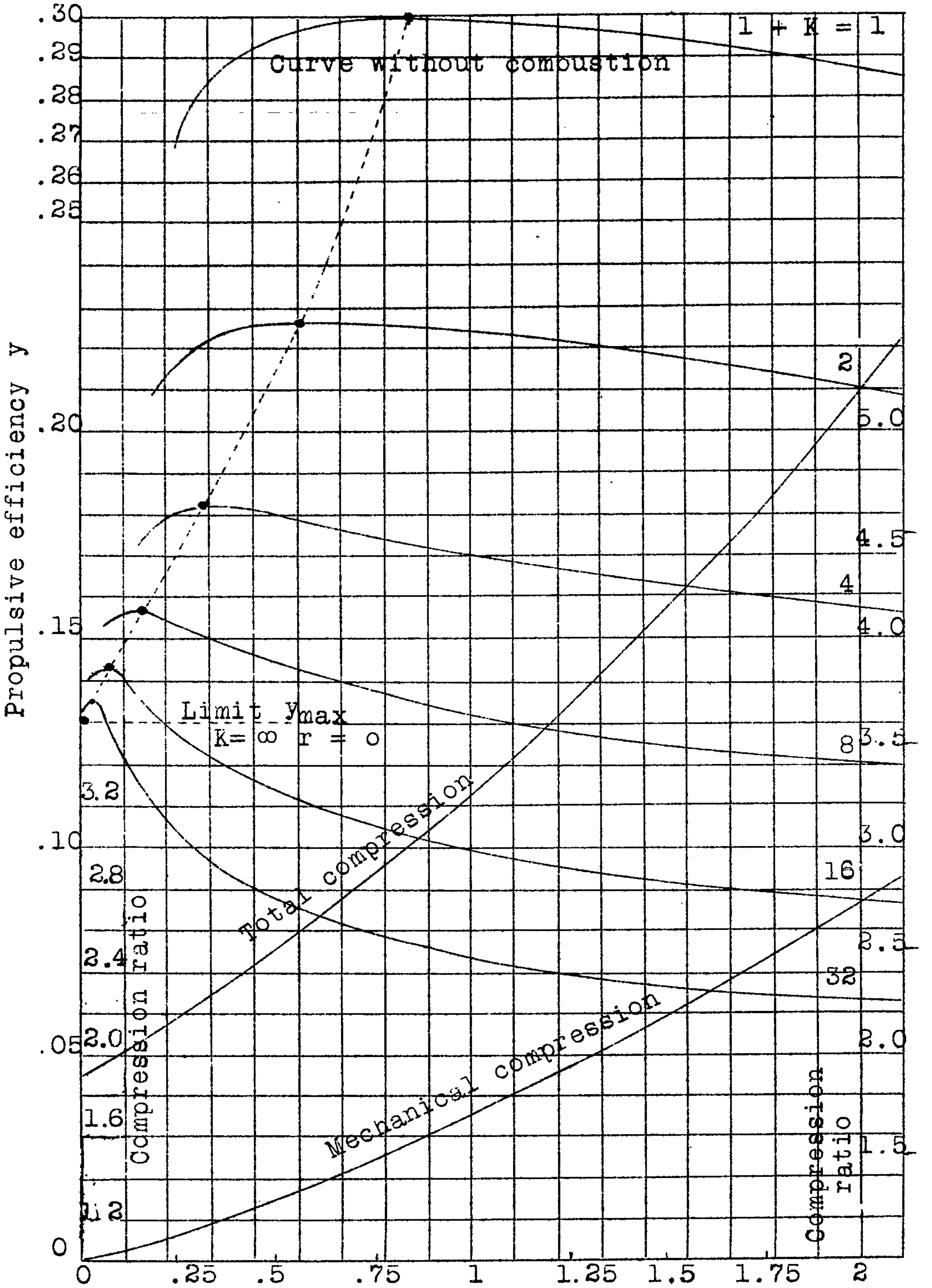


$$r = \frac{\Delta Tm}{\theta \Delta Td} = \frac{\text{Mechanical compression}}{\text{Dynamic compression}}$$

Plot 3.- Temperature.
v = 700 km/h
t = -56.° C.

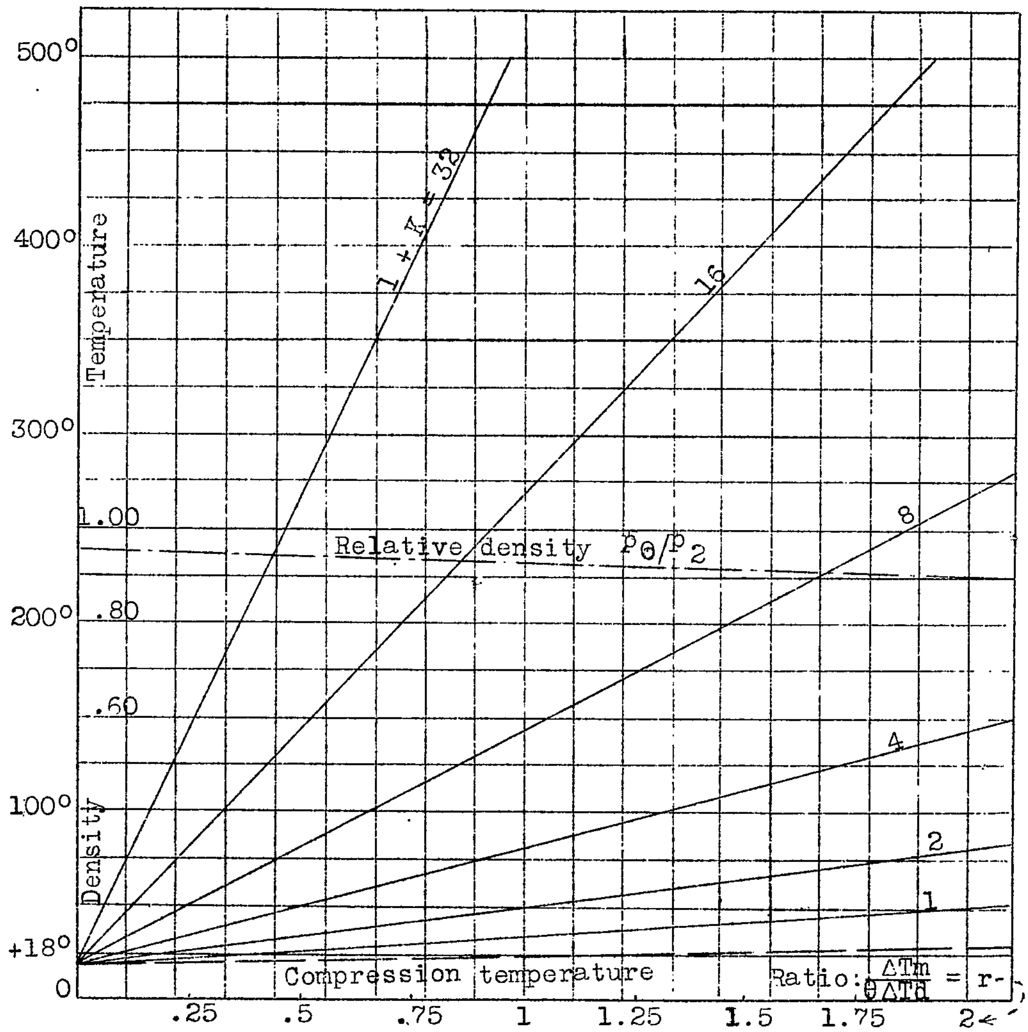


Plot 4.- Temperature.
v = 900 km/h
t = -56.5° C.

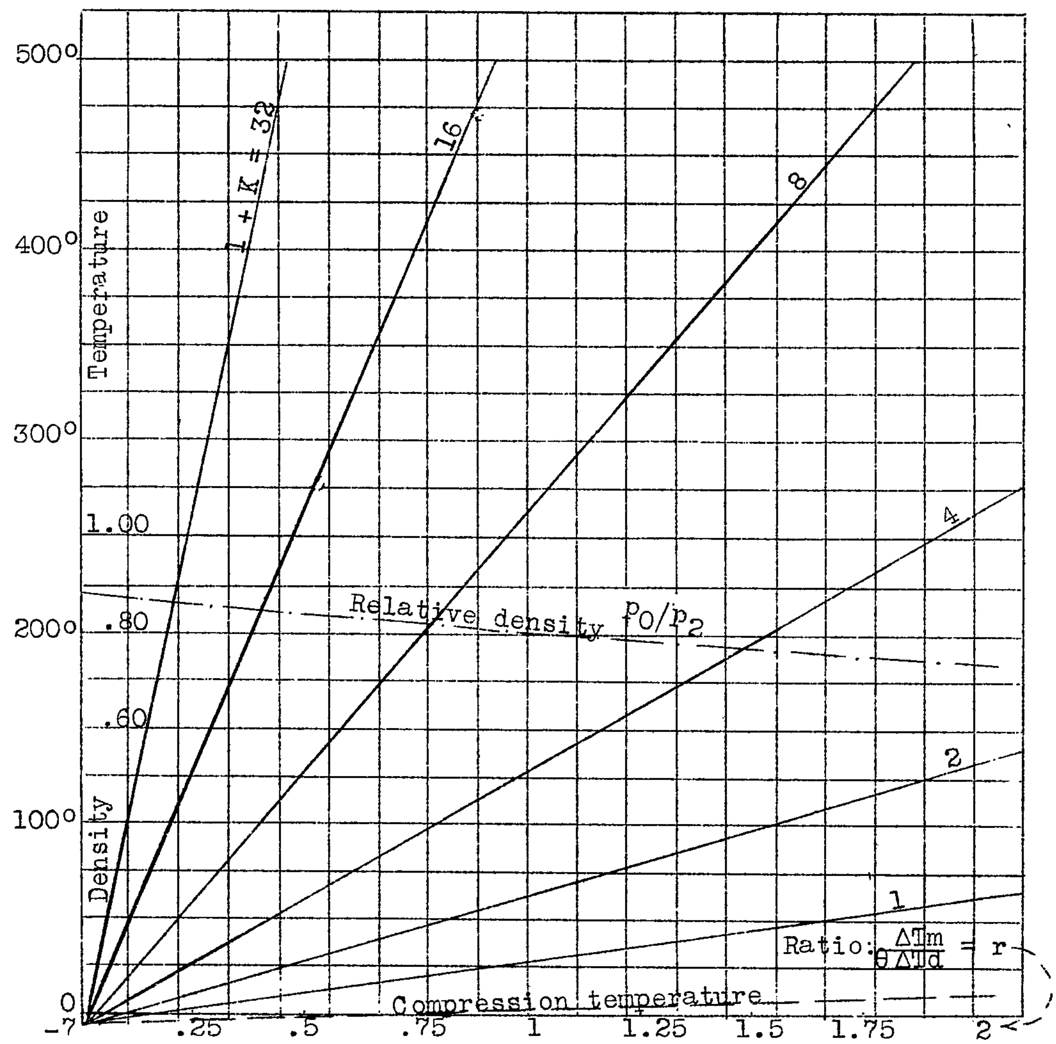


Ratio $\frac{\text{Mechanical compression}}{\text{Dynamic compression}} = \frac{\Delta Tm}{\theta \Delta Td} = r$

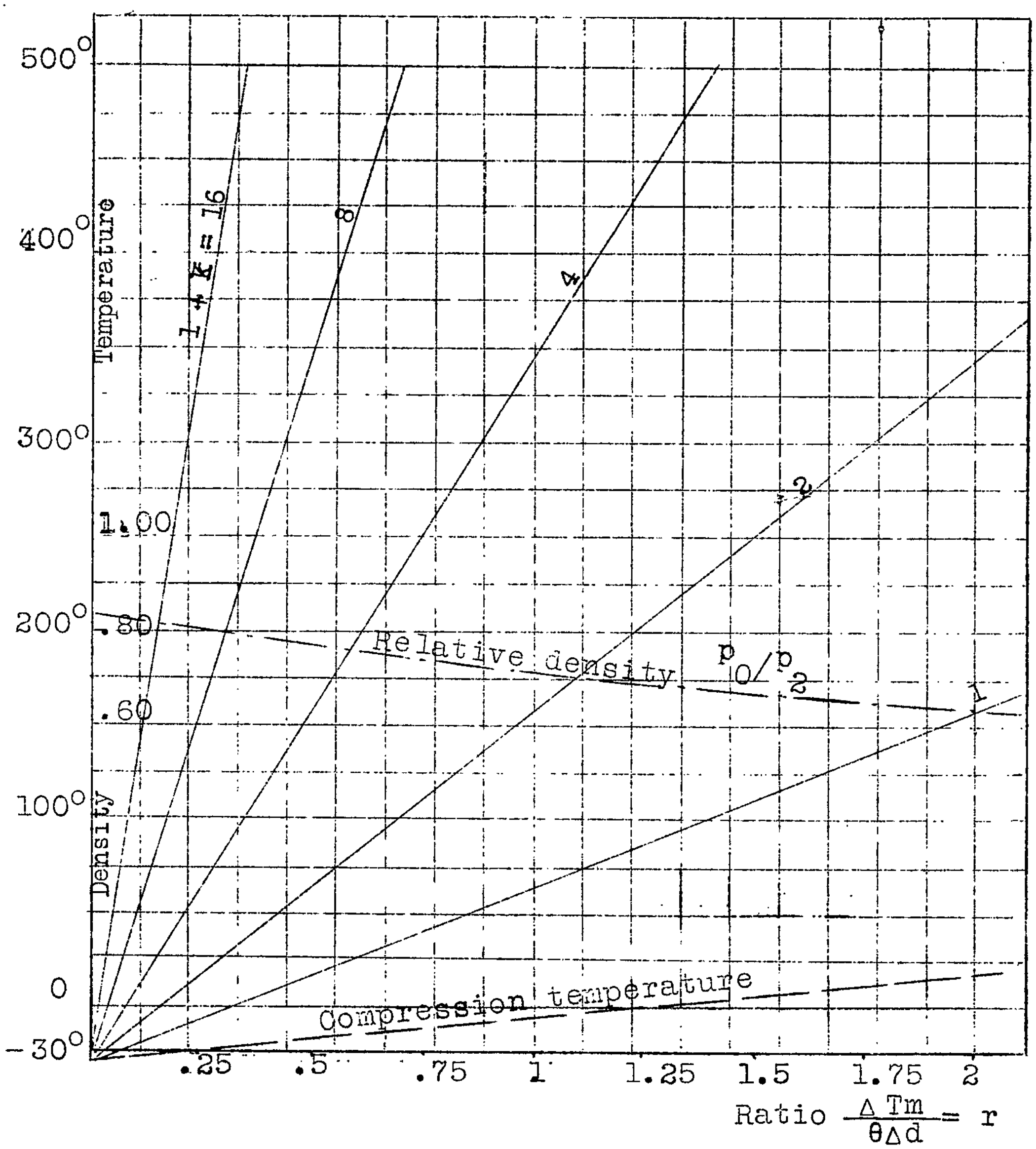
Plot 5.- Temperature.
$v = 1100$ km/h
$t = -56.5°$ C.



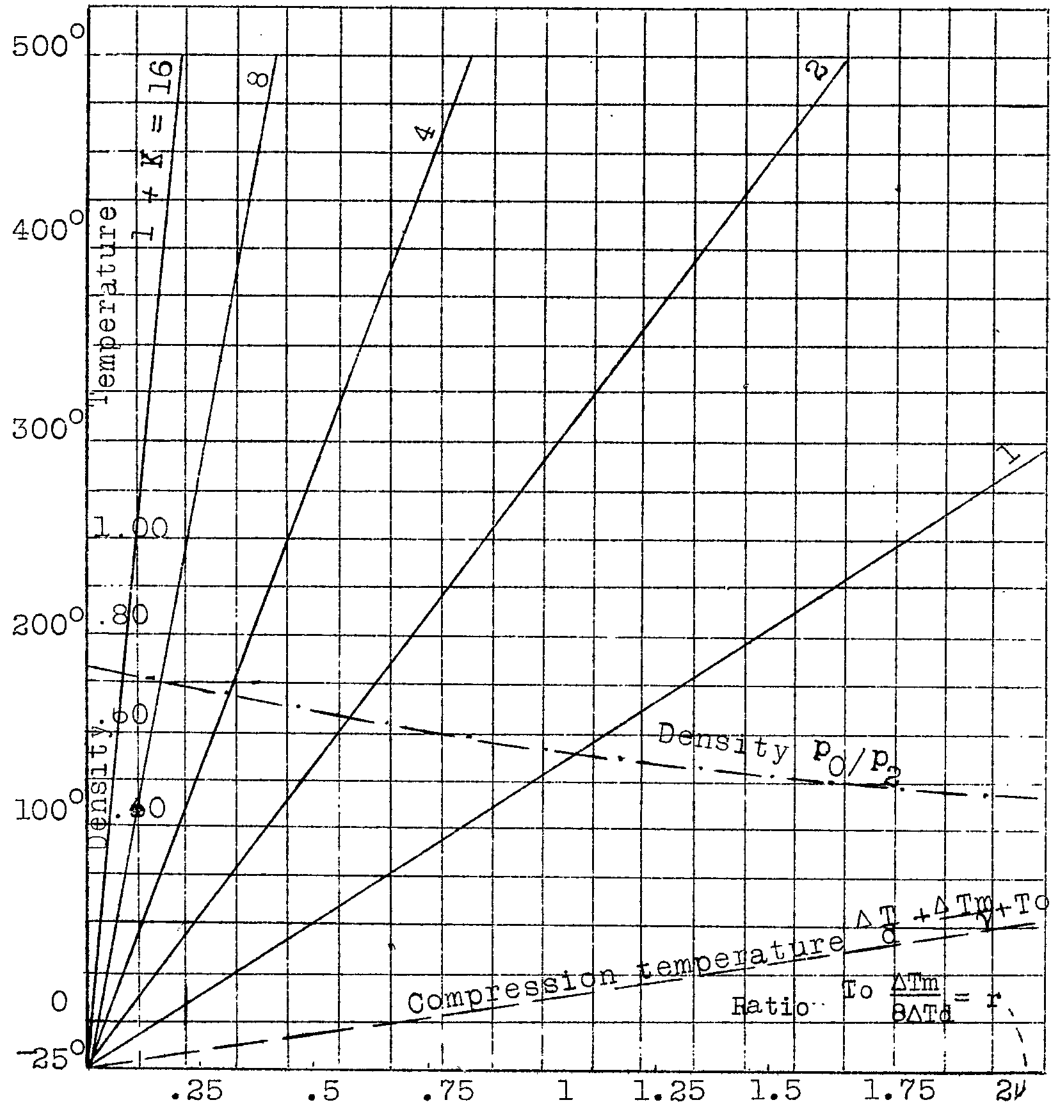
Plot 1A.- Temperature.
v = 300 km/h
t = + 15° C.



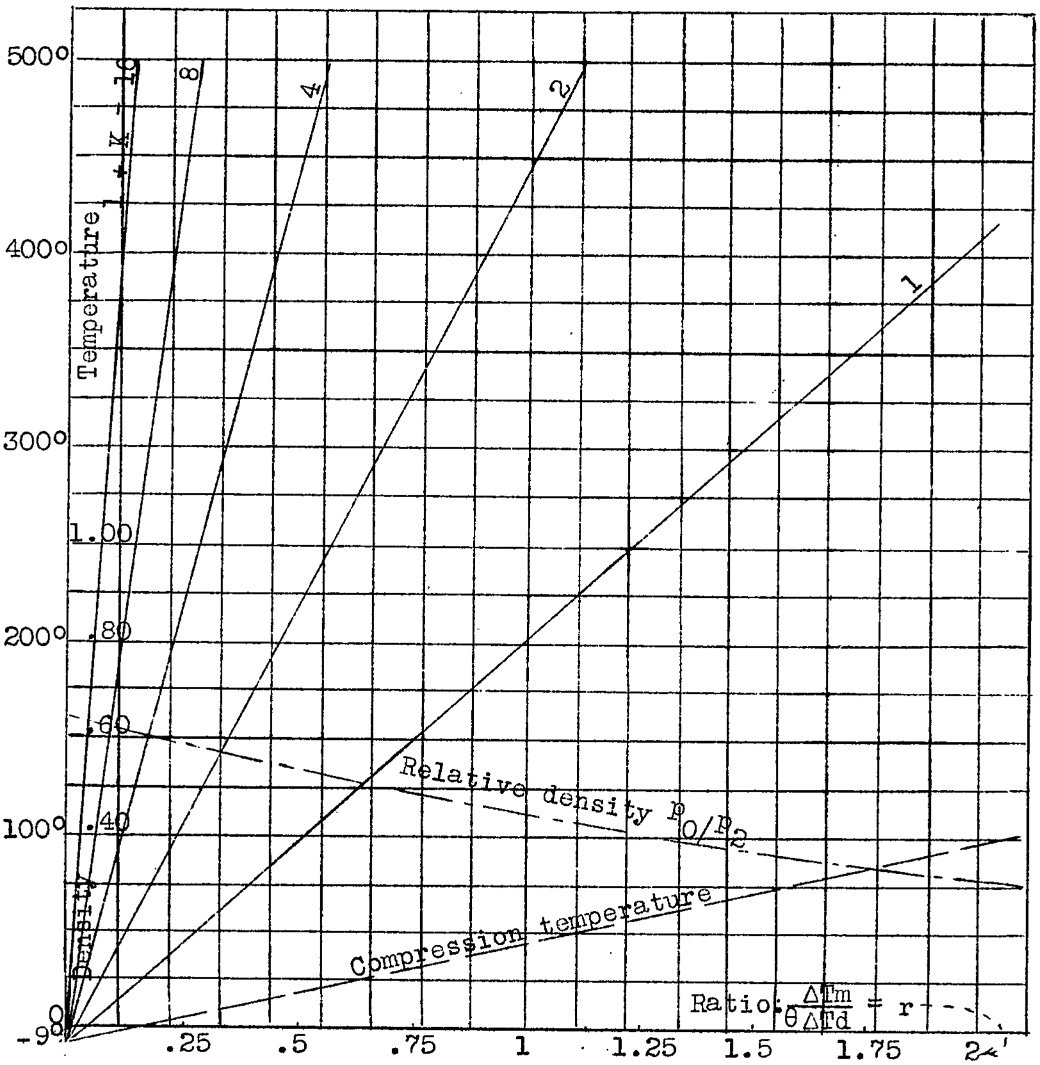
Plot 2A.- Temperature.
v = 500 km/h
t = - 17° C.



Plot 3A.-Temperature.
v = 700 km/h
t = -56.5° C.



Plot 4A.- Temperature.
v = 900 km/h
t = -56.5° C.



Plot 5A.- Temperature.
v = 1100 km/h
t = - 56.5° C.